THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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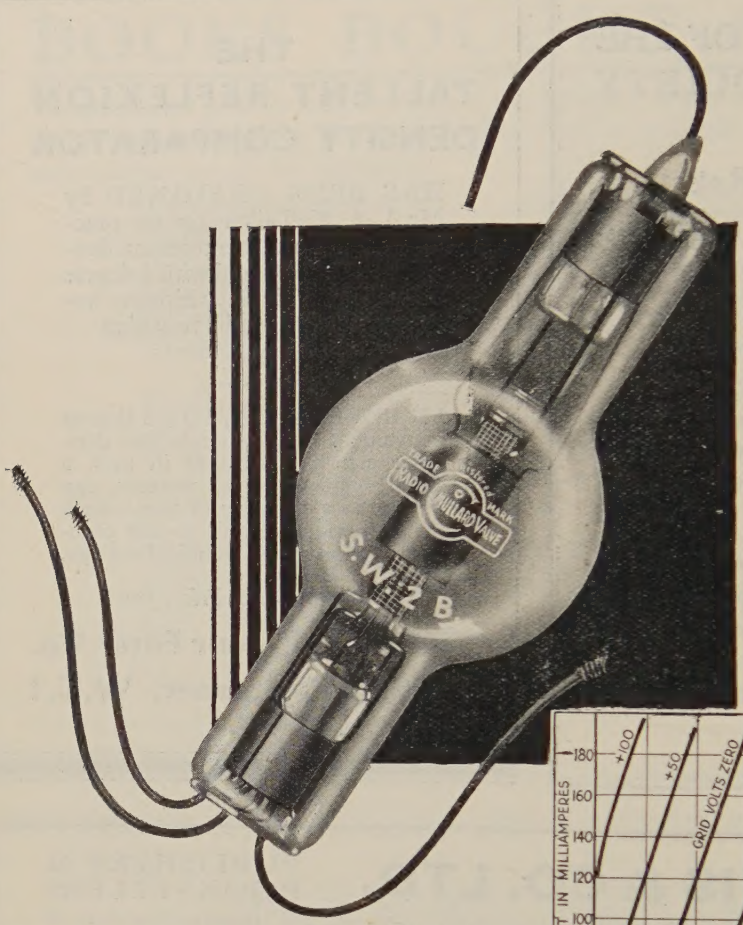

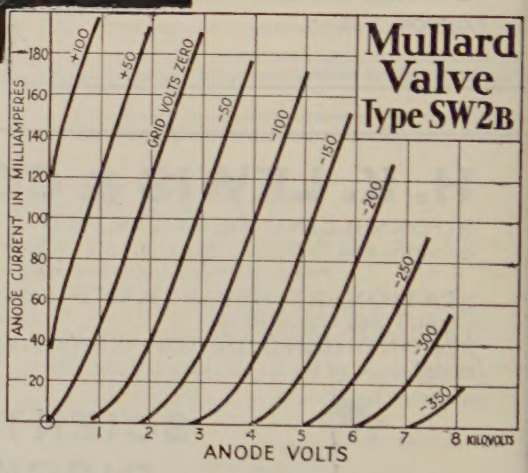

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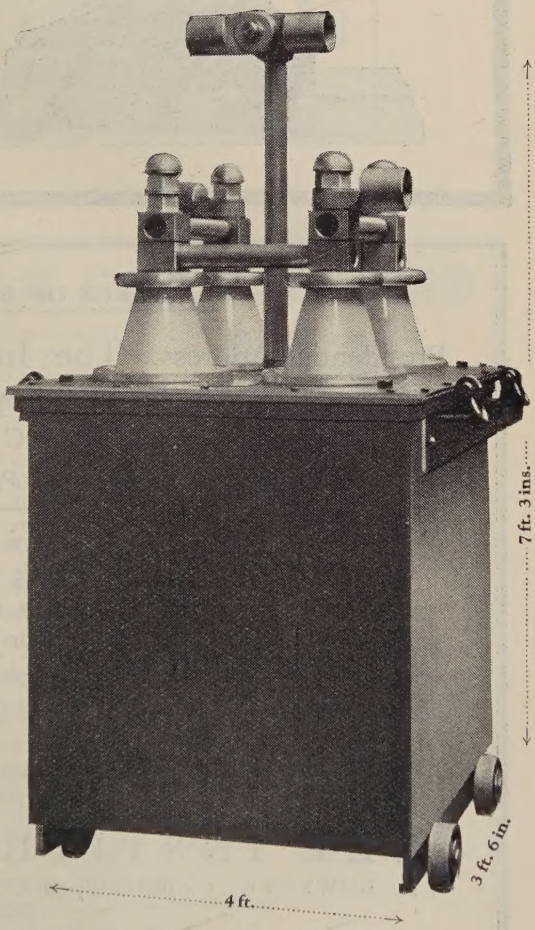

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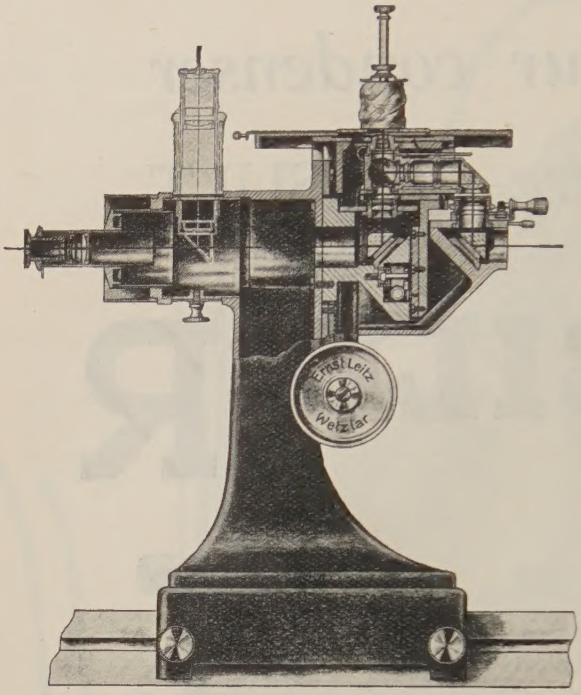

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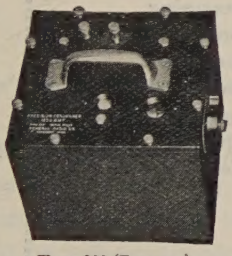

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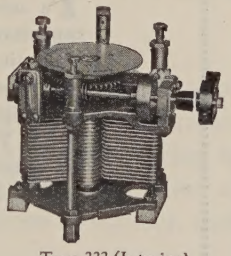

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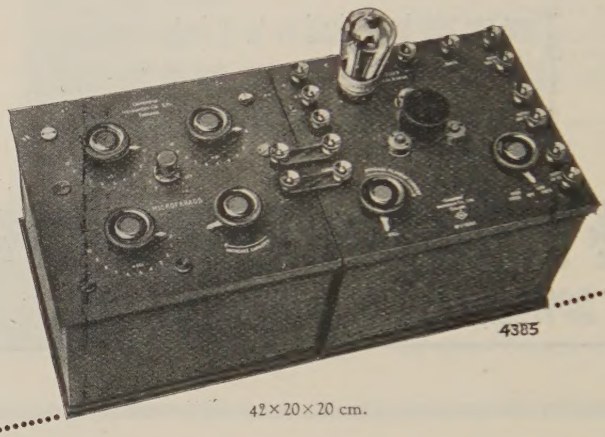

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THE GENERATION OF CURRENT PULSES OF RECTANGULAR WAVE-FORM

BY A. J. MADDOCK, M.SC., A.INST.P.

Received February 6, 1931. *Read April* 17, 1931.

ABSTRACT. A circuit for the production of single current-pulses or for use as a heavy-duty time switch is described in which there is practically no limit to current-carrying capacity. Such pulses of current can be obtained when desired by the simple depression of a key. It is shown that two distinct wave-shapes are available in different branches of the circuit and that by suitable choice of inductance, resistance and capacity the current time wave in one branch should be of rectangular form. Two separate timing-circuits are described, one for periods up to about 2×10^{-2} sec. and the other for use up to 16 sec. or longer. Suggested uses and extensions of the circuit are given.

§ 1. INTRODUCTION

Of the various methods which have been employed for applying a voltage to a circuit for a short interval of time or for obtaining single pulses of current, many are not suitable for currents greater than about 0·5 amp. owing to sparking which may occur at the contact devices and give an increased time of current flow, whilst some mechanical devices, owing to variations in contact, may not be sufficiently accurate.

The circuit to be described was designed to eliminate the current limit referred to and for this purpose a hot-cathode mercury-vapour triode, known as a thyratron, is used as the contact-making and breaking device while, for the obtaining of accurate timing in the duration of current flow, use is made of an electrical oscillatory circuit. The circuit in fact was primarily to constitute a time switch capable of passing from a few milliamperes up to many amperes of current. One of the conditions therefore is that at "make" the current shall rise instantaneously to its full value set by other circuit conditions, remain steady for the duration of flow and fall to zero instantaneously at the point of "break"; in other words, the circuit is to produce single current-pulses of rectangular wave-form.

For a full description of thyratrons the original papers of Hull[1, 2, 3] should be consulted; it will suffice here to say that in operation the mercury is ionized so that the tube becomes practically a space-charge-free device with a voltage drop between cathode and anode of 12–15 volts which is almost constant for any current up to a limit set by the cathode dimensions. Tubes have already been made capable of passing 100 amp. or more. Current flow through the tube can be prevented by a suitable negative bias on the grid, but once the discharge has been initiated the grid has no further controlling action, its effect being neutralized by the positive ions of mercury which collect around it; the discharge can be stopped only by removal of the applied anode voltage or the application to the anode of a momentary negative voltage of sufficient duration to allow of the tube deionizing.

§ 2. CIRCUIT ARRANGEMENT AND ACTION

The circuit diagram is given in figure 1 and can be analysed into three divisions: (1) The main thyratron A with its load resistance R_A and battery B_1. (2) The secondary thyratron B, load R_B and battery B_1. (3) The timing circuit LR_1R_2C in the grid circuits of the valves. It is in the main circuit AR_A B_1 that the current pulses are generated.

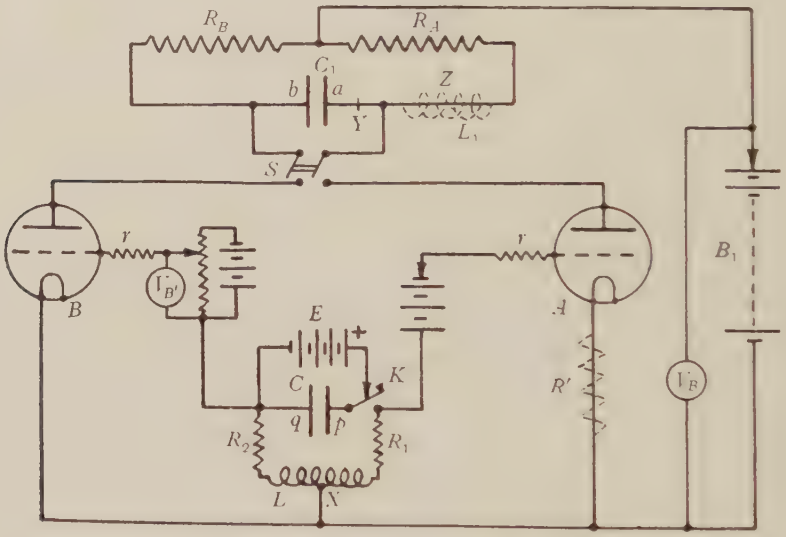

Fig. 1. Circuit diagram for short time intervals.

Deferring for the moment consideration of the timing action, we can consider the action of divisions (1) and (2). Suppose that thyratron A has been released and that current is flowing through it, then if we suppose the battery voltage of B_1 to be 220 volts and the drop across A 15 volts, the plate a of condenser C_1 is at a potential of + 15 volts and plate b at + 220 volts, a difference of 205 volts. (All voltages are referred to the common negative ends of the filaments.) If now

thyratron B is released, plate b is connected to the cathode of A through valve B. Thus plate a, and hence the anode of A, is 205 volts negative with respect to plate b, so that to the anode of A we have, in effect, applied a momentary negative voltage of 205 − 15 (the tube drop of B), i.e. 190 volts; the discharge through A is therefore stopped owing to the negative potential of the anode. Condenser C_1 promptly discharges through B and R_A and charges up in the opposite direction, so that plate a is ultimately at + 220 volts, but, provided the resistance of the load R_A is sufficiently high, it is possible for the tube A to deionize before the voltage of plate a and the anode are sufficiently positive for the discharge to recommence: the functions of condenser C_1 and thyratron B are thus to stop the current flow through A by applying a momentary negative voltage to its anode[3].

The timing circuit, constituting division (3), has to release thyratron A when desired and after the required interval of time to release thyratron B, which then stops current flow through A as already explained. The condenser C is charged by the battery E and when desired the key K is depressed so that the condenser is discharged through the inductance L and resistances $R_1 R_2$, which are of such values as to cause the discharge to be of a damped oscillatory nature. Since the mid-point of the inductance is connected to the cathodes, during the first quarter-cycle negative bias on the grid of thyratron A is annulled by the positive voltage of the plate p of the condenser, so that thyratron A is released and current can flow in its anode circuit; at the same time the normal negative bias of B is increased by the voltage between X and plate q. During the next quarter-cycle, when the voltages are reversed, thyratron B is able to start as soon as the positive voltage between X and q, opposed by the steady bias applied to the grid, is equal to the critical bias*. (A study of figure 2 should make the action clear.) It is arranged that the damping be sufficiently great to ensure that when next p is positive even at the peak of the wave, the resultant bias on the grid of A shall not be reduced to the critical bias: thus thyratron A does not restart, and only the single pulse of current has passed from cathode to anode. Current continues to flow in thyratron B until the switch S is opened; a double-pole switch is necessary to open the anode circuit of A also, otherwise the charged condenser C_1 is liable to release A when B is stopped. C_1 is automatically discharged through R_A and R_B when S is open, and the circuit is now ready to be employed again.

It will be seen that the steady bias of A is not at all critical, and provided the first positive peak of the oscillatory wave is able to reduce it below the critical bias, whilst the second peak is unable to do so, we have the essentials for thyratron A starting once only. It is the conditions obtaining on thyratron B, together with the timing circuit, that determine the time of current flow in A, and in order that the grid voltage of B may not slowly approach the critical bias but undergo a definite

* The critical bias is the bias at which the grid just loses control and anode current commences to flow: it is a function of the anode voltage, a larger anode potential requiring a greater negative bias. The relation is almost linear except for low values of anode and grid potentials. The temperature of the vapour, by altering the pressure within the arc space, also modifies the critical bias and voltage across the tube when current is flowing, and for this reason similar conditions should always be aimed at in the operation of the circuit, especially with regard to the stopping thyratron.

sudden change*, it is arranged for the steady bias to be only slightly in excess of the critical bias, so that B starts soon after the reversal of polarity in the oscillatory circuit: i.e. soon after a quarter-period when the slope of the voltage wave is a maximum.

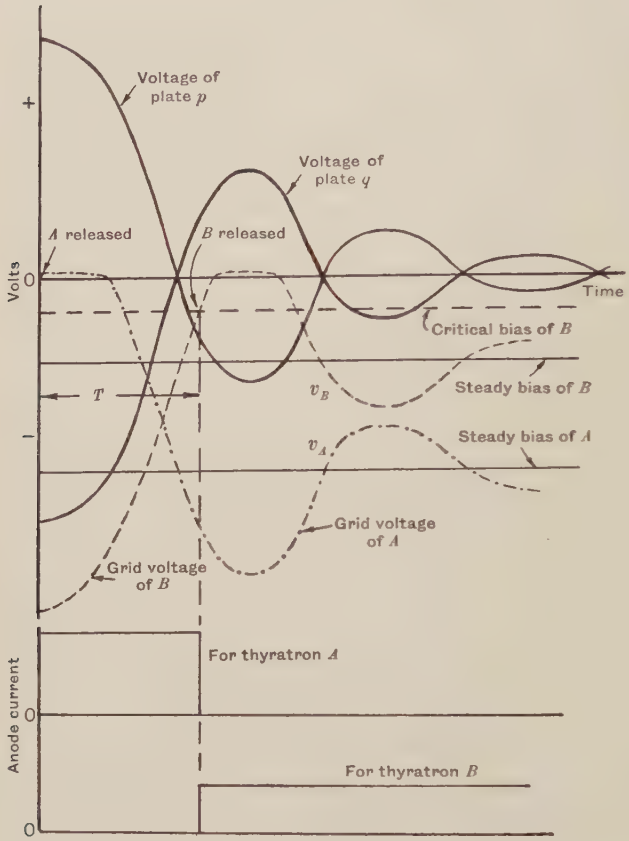

Fig. 2.

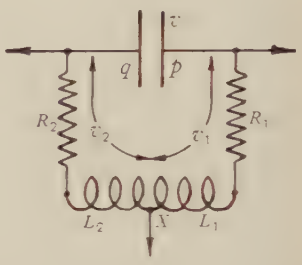

Fig. 3. R_1 and R_2 are the combined resistances of the chokes and any extra damping resistances inserted.

C, E
L, R, v

If the condenser of capacity C charged to a voltage E is discharged through the inductance L and total resistance of the circuit R, then the voltage v across the condenser at any instant is given by

t

$$v = (En/p)\, e^{-kt} \cos(pt - \phi) \qquad \text{.....(1),}$$

n, k, p

where $n^2 = 1/LC$, $k = R/2L$ and $p = \sqrt{(n^2 - k^2)}$, $(n^2 > k^2)$.

If the inductance and resistance were equally divided by point X the voltage either

* Owing to temperature changes noted on page 373 (footnote) it is advisable that a sudden change of bias should occur rather than that it should gradually approach the critical bias, in which latter case changes which occur with temperature considerably affect the bias at which the discharge is initiated.

side would be $v/2$, but as the inductances are not likely to be exactly equal we can facilitate the calculation of the voltages as follows (see figure 3):

$$v/v_2 = (v_1 + v_2)/v_2 = 1 + (L_1\,\partial i/\partial t + R_1 i)/(L_2\,\partial i/\partial t + R_2 i)$$
$$= 1 + L_1/L_2 + i\,(R_1 - R_2 L_1/L_2)/(L_2\,\partial i/\partial t + R_2 i),$$

where v_1 and v_2 are the voltages at any instant across the two halves of the circuit. v_1, v_2
If we make R_1 equal to $R_2 L_1/L_2$ the last term is always zero and then

$$v_2 = vL_2/(L_1 + L_2) \qquad \text{.....(2).}$$

Hence, knowing the inductance values and arranging the resistances as above, we can express the oscillatory voltage on one side in terms of the condenser voltage as given by equation (1). Probably the best and simplest way of arriving at the time of current flow in thyratron A is to draw the oscillatory voltage curve v_2 for the plate q of the condenser on a large scale and to move the time axis up and down in accordance with various selected values of grid bias, in effect obtaining a curve similar to v_B of figure 2. The circuit is arranged so that damping resistances may be inserted if required to increase the natural resistances of the inductances and to satisfy the above relation.

The timing circuit will, of course, operate satisfactorily with an iron-cored choke coil, but it is then not an easy matter to obtain the voltage curve, since the inductance varies both with frequency and with a.c. voltage; for these reasons air-cored chokes and non-inductive condensers are used. An indication of the condenser values required with a 20-henry choke coil (centre-tapped) for various required times of current flow are given in table 1; the time interval is calculated as being one quarter-period of a simple-harmonic oscillation, but owing to damping and the fact that in practice the steady bias must be slightly greater than the critical bias, both factors operate to give longer time intervals than those indicated.

Table 1. Approximate values of capacity for various times.

Time (sec. $\times 10^{-3}$)	L (henries)	C (μF)
1	20	0·02
2	20	0·08
5	20	0·5
10	20	2·0
20	20	8·0

The circuit has operated very satisfactorily at all these values and it should be possible to obtain times shorter than 10^{-3} sec. by the use of a smaller inductance. Resistances r, figure 1, of 0·25 megohms are placed in each grid lead to prevent large electron-currents from flowing in the grid circuits when the grids tend to become positive.

§ 3. EXPERIMENTAL RESULTS

For the purpose of observing the shape of the current time curves in the main thyratron circuit a cathode-ray oscillograph was used with which the transient could be seen visually, but in order to obtain a diagram of the wave-shape the timing circuit as described was removed and the grids driven by a.c. at 50 ~ from a centre-tapped transformer. A continuous series of pulses was obtained in each circuit instead of only the one used normally; sketches only were made of the wave-shape on the screen and are given herewith.

Owing to the charge and discharge currents through the stopping condenser C_1 the current pulses will be of different shape in the anode and cathode leads of thyratron A. We shall consider first of all the wave-shape in the anode lead, i.e. the wave-shape of the current flowing through the resistance R_A.

(*a*) *Current in anode resistance.* With a pure resistive load the wave-shape is as shown in figure 4 (*a*), the current commencing suddenly, having a flat top and at the end a peak and exponential decay due to the discharge and charge in the reverse direction of condenser C_1. The sketches are drawn approximately to the same scale, and the "true time" of current flow, as it may be called, was 0·01 sec. in all cases, this being the time between the release of thyratrons A and B.

This peak at the end was an undesirable feature and to reduce it a choke of low d.c. resistance was placed in series with condenser C_1 in position Y. Results with various values of inductance are given in figure 4 (*b*) (*c*) (*d*) (*e*) whence it will be seen that the peak has been removed, and for some classes of work the wave-shape of figure 4 (*d*) might be suitable. Another method of eliminating the peak was to place a condenser across the stopping-thyratron B with the choke as before, for which it was found that a lower value could be used, see figure 4 (*f*): this method is not advisable as high peak currents are set up by the discharge of this auxiliary condenser directly through thyratron B, with possible damage to the cathode. The wave-shapes with different values of inductance in the common d.c. supply lead are given in figure 4 (*g*) (*h*) (*i*) (*j*), (*h*) being fairly symmetrical and flat topped. Various other attempts were made to annul or modify the condenser current, including the use of a double circuit of two thyratrons in series of which one was used purely for stopping the current flow; but even in this case condenser current (though of a smaller magnitude) did still flow through the load since this formed, in effect, a parallel circuit.

Although with the methods tried we have been unable to obtain rectangular pulses in the anode resistance, yet for certain work the waves obtainable therein may be useful and it can be seen how the contour may be modified if desired.

The circuit was originally intended as a short-time switch for measuring total emission of oxide-coated cathodes in thermionic valves. The valve under test was to constitute the load, being inserted in place of the resistance R_A in figure 1 so as to afford, together with the voltage applied by the battery B_1, the only limit to the current. The nearest approach to sudden application of voltage for a short period was obtained by the circuit conditions that gave the current curve of figure 4 (*d*).

A ballistic galvanometer can be used to measure the total quantity passed, and thence the value of the current can be obtained from a knowledge of the time of application of the battery voltage. It is possible to calibrate the galvanometer to read current directly without the time interval being calculated, but as this interval can easily be found from equations (1) and (2) the method described above seems preferable. One of the chief drawbacks to a thermionic valve constituting the load is that since different valves have different effective resistances, when these are inserted in turn as the load the time of charging of condenser C_1 will be variable, and in some cases may be so short as not to allow thyratron A to deionize; insertion of an extra anode resistance is a method of overcoming this difficulty, though a considerable voltage will be lost across the resistance for large currents.

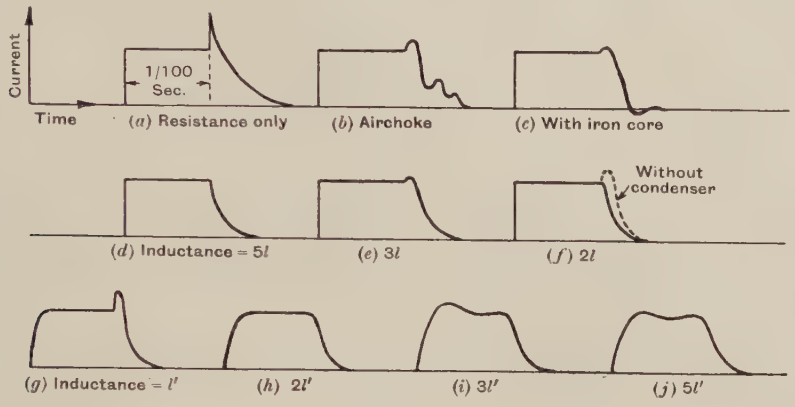

Fig. 4.

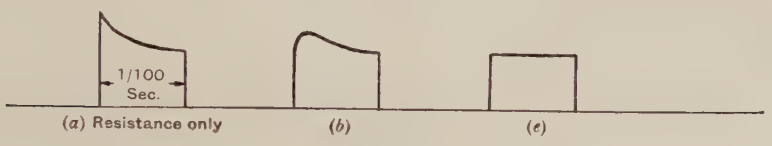

Fig. 5.

(*b*) *Current in cathode lead.* We now consider the conditions in the cathode lead of thyratron A. Current starts and stops suddenly with resistive loads, and the only additional current is that which charges condenser C_1 to the voltage drop across the anode resistance when A is released; this gives a wave-form as shown at figure 5 (*a*): the insertion of a choke at Y rounds off the peak and gives a wave of the shape shown in figure 5 (*b*). The advantage of working in the cathode lead is that the wave has perpendicular ends, though it must be borne in mind that an anode resistance must always be used to limit the current.

It is much regretted that circumstances have not permitted of the author's carrying out further experiments on the wave-shape in the cathode lead, but from the following consideration it will be seen that rectangular pulses should be obtained if the circuit as given in figure 1 were modified by the insertion of an inductance L_1

at Z. When A is released inductance L_1 and resistance R_A are in parallel with condenser C_1; hence the inductive growth of current in the cathode lead due to L_1 and the capacitative charging current of condenser C_1 can, together, be made to give a steady current in the cathode lead from the commencement of flow, the strength of this current being equal to the final strength reached. I.e. the pulse will be flat-topped, figure 5 (*c*). It is easy to show that for this condition to be realized R_A must equal R_B (R_A now including the d.c. resistance of inductance L_1) and

R_A

L_1

$$L_1 = C_1 R_A^2 \qquad \text{.....(3).}$$

Thus C_1 and R_A may have practically any value and the wave-shape may still be rectangular provided a suitable value of choke is inserted in the circuit to satisfy equation (3). The obvious arrangement is to make R_A small to avoid a large voltage drop across it with correspondingly increased value of C_1. The limiting factor is again the time of deionization of thyratron A.

We shall now attempt to fix limits for the various circuit values imposed by consideration of this deionization-time. Although the grid bias of A when the arc current ceases gradually becomes increasingly negative as the positive ions are neutralized, yet if ions are present in the arc space a discharge could be again initiated when the anode voltage is slightly greater than the normal voltage drop which obtains when current is flowing. For convenience of working and to obtain a general idea of the circuit conditions it will be assumed that if ions are still present the arc can recommence at the same voltage as the normal tube drop of E_c volts. If the time of deionization is T_0 sec., the aim in fixing the values of the circuit components must be to arrange that the voltage between cathode and anode of thyratron A shall not, owing to the charging up of condenser C_1, reach a value E_c volts within the time T_0. A further simplification is to assume that the tube drop of thyratron B when passing current is also E_c; these simplifications are justifiable since we are concerned only with setting a limit to the values of C_1, R_A, etc., so that thyratron A is released once only. If E_b is the battery voltage, then when B starts the p.d. across the condenser is $-(E_b - E_c)$, which finally becomes $+(E_b - E_c)$ when the condenser has discharged and been charged in the reverse direction. Thyratron A can start again, if positive ions are present, when the p.d. across C_1 is zero; this takes place, in the limit which we are considering, in time T_0 after B is released. The problem is therefore the same as that presented by a condenser initially with zero charge and finally charged to a value double that which it has at time T_0. The above considerations hold for the circuit however it may be modified to alter the wave-shape. With the choke L_1 inserted to make the form rectangular it will be noticed that, when B is released, condenser C_1, inductance L_1 and resistance R_A are all in series, and since $L_1 = C_1 R_A^2$ the current tends to be of an oscillatory nature. It can be shown that at time T_0

$$(\sqrt{3}/4) \exp(T_0/2C_1R_A) = \cos(T_0 \sqrt{3}/2C_1R_A - \pi/6) \qquad \text{.....(4).}$$

T_0 increases with increase of arc current due to the greater number of positive ions, but for any one value of current it is to be noticed that, on this simplified working,

T_0 is independent of the battery voltage, the explanation being that although with a higher battery voltage the condenser charges up more rapidly initially, a lower negative voltage was originally applied to the anode of thyratron A when B was released.

The minimum values of C_1 and R_A for which thyratron A is just not released a second time, having been determined experimentally for a given set of circuit conditions, T_0 can be calculated from equation (4). If now it is desired to use a different value of R_A, the values of C_1 and L_1 can be found for the same current for which T_0 will be the same. For example, for a certain current, with the thyratron used it was found that T_0 was approximately 200 microseconds. For R_A to equal 50 ohms, substitution in (4) gave a value for C_1 of $3{\cdot}1\,\mu$F with a consequent value 0·0078 H. of L_1 as determined by (3) to keep the current pulse flat-topped. These are the limiting values and slightly greater ones would be chosen in practice. It will be seen that heavy currents can be passed when the circuit is used in this manner with a moderate anode voltage supply.

If the circuit in the cathode lead into which we wish to inject the current (i) has a resistance R', then the negative voltage applied across thyratron A when B starts is

$$-R_A i + E_c = -[E_b - (E_c + R'i)] + E_c.$$

If this is less than E_c, the discharge in A is stopped: i.e. for A to be stopped at all

$$E_c > -[E_b - (E_c + R'i)] + E_c,$$

whence
$$E_b > E_c,$$

since
$$i = (E_b - E_c)/(R_A + R').$$

Thus the minimum battery voltage to be used is equal to the normal tube drop, no matter what values R_A and R' may have or what the current value is. In practice it must be greater than this, for E_b must always be greater than the value required to start the arc in A when its grid bias is reduced below the critical bias by the timing circuit. It should be noted that there must always be a resistance R_A to give a voltage drop across the condenser for stopping A.

By working in the cathode lead we have therefore achieved the object of obtaining a single pulse of current of rectangular form and, further, the circuit is more flexible than when the current in the anode resistance is used, for it has been shown that neither the voltage of the anode supply nor the value of the resistance in the cathode lead affect the time of deionization, except in so far as they determine the magnitude of the current. If, therefore, R_A, L_1 and C_1 were kept the same and set for the largest current, then a large range of current values less than the maximum could be obtained by alteration of the battery voltage E_b without trouble arising over variation of the time of deionization with the arc current. Thus under these conditions the circuit is well adapted to the measuring of total emission of radio valves inserted in the cathode lead and constituting the resistance R'. Other suggested uses are the testing of fuses, cut-outs, circuit-breakers, overload relays, etc., when a known extra current could be applied to them for a known length of time, it being thus possible to determine the time delay in their action and the percentage overload carried

during short time intervals. The circuit, in fact, constitutes a heavy duty time switch and can be used wherever such an arrangement is required. It is possible, if desired, to obtain two or even three pulses of current all of equal magnitude through A by reducing the damping of the oscillatory timing circuit and decreasing the steady bias of A towards the critical bias.

§ 4. EXTENSION OF THE CIRCUIT

The circuit described above is essentially designed to obtain single pulses of current of rectangular wave-shape of any magnitude and duration. By suitable design almost any shape can be given to the current wave, including a practically sinusoidal form. Prince[4] has constructed a self-oscillatory circuit in the anode circuits of the thyratrons so that when these are fed with d.c. an alternating current is obtained in the output circuit; this can, if required, be stepped up or down, rectified and smoothed, forming a complete inverter circuit or "d.c. transformer." The grids can be driven by any alternating voltage instead of that derived from the damped oscillatory circuit described above, and it occurred to the writer that the grids might be controlled by a tuning fork maintained in vibration by the output, the power developed in the anode circuit being used to control the frequency of a wireless transmitter; greater power is obtainable from these two valves alone than by many stages of amplification with vacuum valves or by the use of fork-controlled flashing neon tubes[5]. However, the frequency with which the thyratrons may be released and stopped has a limit set by the time required for deionization of the space between cathode and anode, but harmonics of this fundamental could be used for higher frequencies.

§ 5. CIRCUIT FOR LONGER TIME INTERVALS

In the circuit of figure 1, for times greater than about 0·02 sec. ($L = 20$ H., $C = 8\mu$F), the size of inductance and condenser become unwieldy and some other timing device must be employed. A suggested circuit is shown in figure 6 which differs from figure 1 only in respect of the method of releasing thyratron A and then B after the required interval; according to figure 6 a condenser is charged through a high resistance and then its partial discharge through a neon tube is permitted in the usual fashion[6]. The action of the circuit is as follows: on closure of the switch K the battery B_2 commences to charge condenser C through the high resistance R_1. The current at the start is a maximum, and flowing through the resistance R_2 from p to q it causes an e.m.f. to be developed across that resistance in opposition to the steady bias on the grid of thyratron A, which is thus released at the moment when switch K is closed. When C is charged to the striking voltage of the neon tube N it discharges through this latter and resistance R_3, thus decreasing the bias of thyratron B, releasing it and stopping A as in the original circuit. The discharge current also flows through R_2 from q to p, increasing the bias of A. When the voltage at which the neon tube discharge can no longer be maintained is reached, the battery commences to charge condenser C again but, as the latter was not fully discharged, the

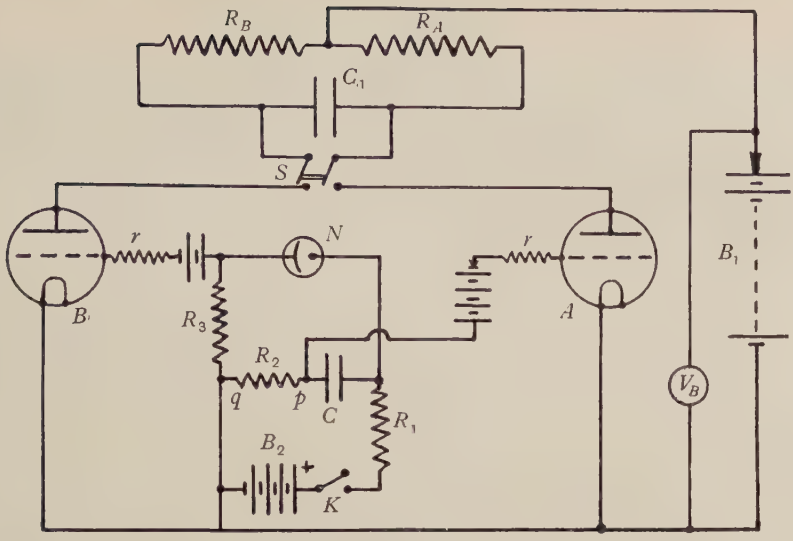

Fig. 6. Circuit diagram for long time-intervals.

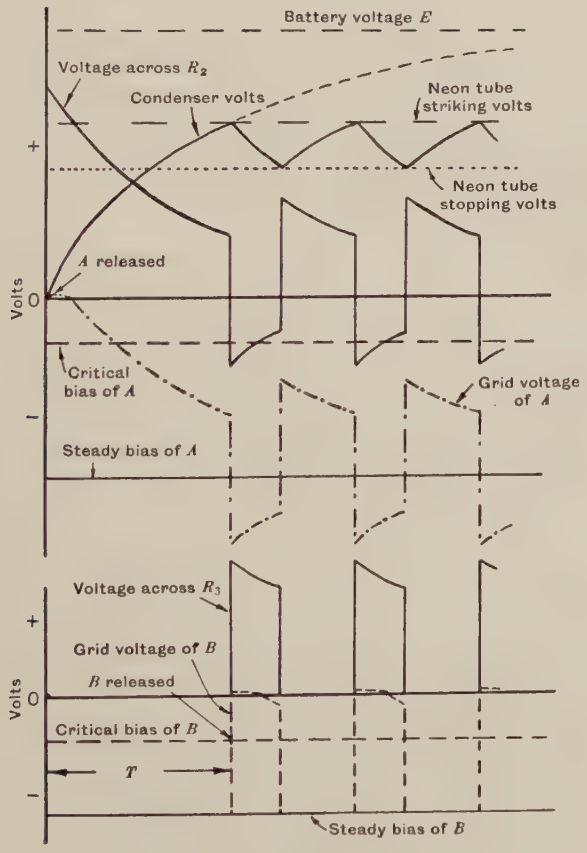

Fig. 7.

current is less than in the first instance and by suitable choice of resistances it can be arranged that the e.m.f. across R_2 shall be insufficient to annul the bias on thyratron A to the critical bias, with the consequence that A is not again released. Thus one pulse will be obtained through thyratron A though B may continue with the neon tube flashing if K is kept depressed. To obtain further pulses we must start over again with C fully discharged. The action should be clear from a study of figure 7.

The time of current flow in A is the time from closing switch K to the commencement of discharge of C through the neon tube. Hence if E is the voltage of battery B_2, V the striking voltage of the neon tube, R the total resistance of the charging circuit and C the capacity, then the time of current flow T is given by

$$T = CR \log_e [E/(E - V)] \qquad \ldots\ldots(5).$$

Times overlapping with those of the circuit of figure 1 and up to many seconds' duration can be obtained by this method, e.g. with E equal to 180 v., V to 140 v., R to 0·1 megohms, and C to 0·1 μF, $T \doteq 0{\cdot}015$ sec.; and with E equal to 160 v., R to 2 megohms, C to 4 μF, $T = 16{\cdot}6$ sec.

§6. ACKNOWLEDGMENT

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REFERENCES

(1) A. W. Hull. *Trans. A.I.E.E.* **47**, 753 (1928).
(2) A. W. Hull. *General Electric Review*, **32**, 213 (1929).
(3) A. W. Hull. *General Electric Review*, **32**, 390 (1929).
(4) D. C. Prince. *General Electric Review*, **31**, 347 (1928).
(5) W. H. Eccles and W. A. Leyshon. *Electrician*, **97**, 65 (1926).
(6) S. O. Pearson and H. St G. Anson. *Proc. Phys. Soc.* **34**, 204 (1922).

AN IMPROVED METHOD FOR THE COMPARISON OF SMALL MAGNETIC SUSCEPTIBILITIES

By R. A. FEREDAY

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ABSTRACT. In continuation of an earlier investigation, it is shown that a magnetic field, specially suitable for determinations of relative susceptibility by an improved non-uniform field method, can be produced by an electromagnet whose pole-pieces are respectively plane and spherically concave.

§ 1. INTRODUCTORY

In an earlier communication* (subsequently referred to as "the previous paper"), an account was given of a new method, of the non-uniform field type, for the comparison of small magnetic susceptibilities. A theory was developed which indicated that between two suitably shaped pole-pieces of an electromagnet a field could be produced having such a configuration that, over a wide region, the product of field-strength by its gradient along the axis of symmetry would be constant, so that a feebly magnetic body placed on this axis would experience a force which would be directed along the axis and constant at all parts of it. Equations were developed theoretically to determine the shape of the necessary pole-pieces; however, as is to be expected in a calculation of this nature, the field-distribution obtained in practice departs somewhat from that predicted by theory.

In order to be able to realize more accurately a field-distribution of the required configuration, it has been necessary to make empirical corrections to the results of the theoretical investigation, and the present paper gives an account of experiments made with this object. These experiments are described in some detail, in the hope that the results may be of use to others faced with the problem of designing an apparatus for susceptibility measurements.

It should be emphasized that the expression "magnetic force" is used to denote the actual force exerted on a weakly magnetic body, this being proportional to the product of the field-strength and field-gradient.

§ 2. EXPERIMENTAL

In the experiments to be described, the field was explored by direct measurement of the force exerted on a small paramagnetic body. Arrangements were made so that the force could be determined at different points along the axis of symmetry of the pole-pieces, or along any vertical line passing through this axis. The actual determination of the force exerted on the body was made with a torsion balance similar in principle to that used for susceptibility-measurements and described in the previous paper.

* *Proc. Phys. Soc.* **42**, 251 (1930).

The torsion balance. This was very similar to that illustrated in figure 4 of the previous paper, but of somewhat simpler construction. The two arms were made of equal length, and observations of deflections made directly on the specimen, which consisted of a small quantity of a strongly paramagnetic salt contained in a glass bulb, of diameter about 2·5 mm., fused to one end of the torsion balance arm.

In the case of the balance previously described, a small correction had to be applied for a deflection due to the magnetic force exerted on the arm of the balance itself. In the present case, an improvement has been effected: the torsion balance arm has been made of a glass whose susceptibility is so small that this effect is negligible.

Ordinary soda glass, such as that used for the construction of the earlier balance, is diamagnetic, having a mass-susceptibility of about $-0{\cdot}2 \times 10^{-6}$. On the other hand, paramagnetic glasses also are available; for the present work some bottle glass having a susceptibility of about $+2{\cdot}0 \times 10^{-6}$ was obtained. This was finely powdered, mixed with some finely powdered soda glass, and fused up in a furnace. The resulting product, which had a susceptibility of $+0{\cdot}8 \times 10^{-6}$, was again powdered, and fused up with more soda glass. In order to secure thorough mixing of the components, the resulting melt was again finely powdered, well mixed, and fused up. By this procedure a glass of extremely small susceptibility (less than 10^{-8}) was obtained. This was fused and drawn into thin rods by insertion into the crucible when the contents were in a viscous state of a stout platinum wire, which was slowly withdrawn together with the adhering glass. In this way, by suitable choice of the temperature and speed of withdrawal of the wire, rods several feet long and of any desired diameter could be produced. One of these rods was selected, and from it the arm of the torsion balance was constructed.

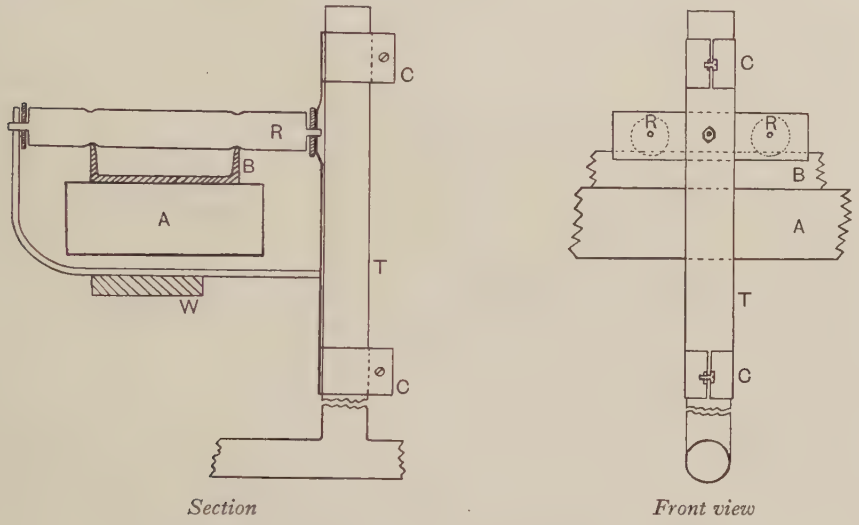
The susceptibility of such a complex mixture may be expected to show a temperature-variation, but since in the present investigation all observations were made at room temperatures no appreciable susceptibility-variation occurred.

The torsion balance support. In order to plot out curves of the force-variation along and perpendicular to the axis of symmetry, it was necessary to be able to move the balance in either of these directions. To this end the T-tube was mounted in clamps in which it could be moved vertically (for curves along a perpendicular to the axis) and the clamps were supported from a carriage which could traverse the magnet horizontally (for curves of distribution along the axis).

A stout brass bar *A*, figure 1, was bolted across the top of the magnet (cf. figure 3 of the previous paper), and on this was supported a piece of channel brass *B*, the upper edges of which were filed down carefully and rounded off to form a pair of rails lying parallel to the axis of the magnet. On these rails could be moved a brass trolley consisting of a brass framework carried on a pair of rollers *R*, in which grooves were turned so that the carriage would roll accurately on the rails. Two brass clamps *C* were rigidly attached to the framework, and held the torsion-balance T-tube *T*. A heavy lead weight *W* balanced the whole and kept the carriage steady on the rails.

Method of observation. The force on the specimen was evaluated by a direct observation, by means of a travelling microscope provided with an eyepiece scale, of

the deflection of the bulb when the magnetizing current was switched off. In order to follow the bulb as it was shifted to different positions along the magnetic axis, or along a perpendicular line, the microscope was arranged to travel horizontally when observations of the force-distribution along the axis were to be made, and vertically for observations along the perpendicular direction. The distance of the specimen from a fixed point was first observed by means of the vernier scale of the travelling microscope, the current was then switched off, and the resulting deflection was measured with the eyepiece scale. In the case of curves of force-distribution along the axis, the mark with respect to which the position of the specimen was determined was a small projection from the centre of one pole-piece; in the case of the perpendicular curves the scale illustrated in figure 3, and fully described later was employed.



Section *Front view*

Fig. 1. Diagrammatic view of torsion balance carriage.

Special arrangements were necessary to ensure accurate setting of the specimen. Figure 2 shows the device used to set the specimen into the vertical plane through the axis of the magnet. The two arms *ab*, *a'b'*, projected horizontally to equal distances from the centres of the poles, and a length of thread was stretched between *a* and *a'* so that it ran parallel to the axis of symmetry of the magnetic system. The specimen was adjusted to a distance from this thread equal to the length *ab*, by means of a depth gauge supported horizontally. This ensured its being on the axis.

Two pieces of card, graduated in millimetres, were attached to the pole-pieces so as to lie in a vertical diametral plane, figure 3. These served as a check on the accurate centring of the specimen, since when situated on the axis it was midway between them.

When observations of the distribution of force along a vertical line perpendicular to the axis were to be made, the microscope had first to be set up to travel in a direction accurately perpendicular to the axis. To this end it was so arranged that

on being displaced vertically, it focussed on the upper and lower scales successively at the same graduations, i.e. at the same distance from the face of the pole-piece. The effect of a small error in this adjustment on the results is discussed in § 3. When

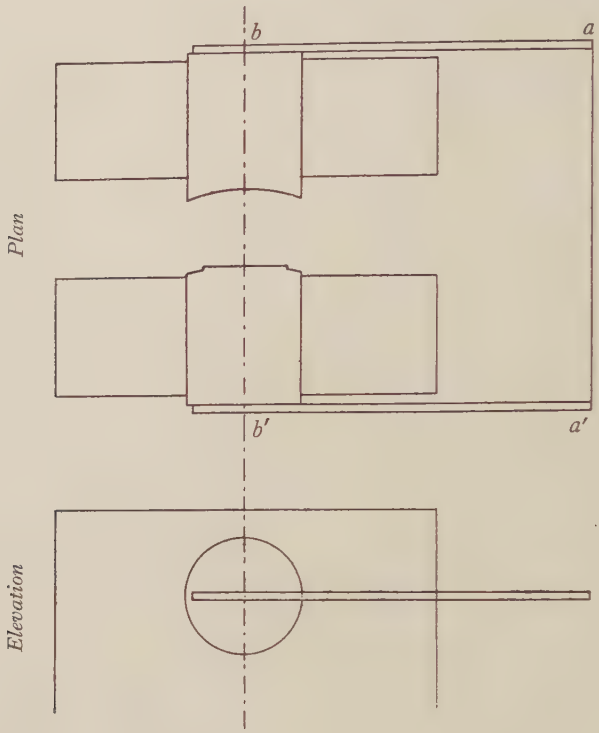


Fig. 2.

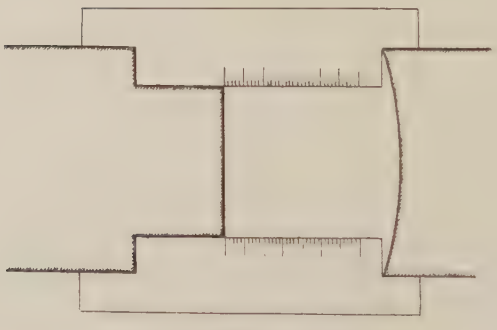


Fig. 3.

observations for these "perpendicular" distribution-curves were being made, the distance of the specimen from the axis was evaluated by an observation of its vertical distance from the side of one of these cards.

§ 3. RESULTS

For the first tests a pair of pole-pieces was constructed to the design of equation (8) of the previous paper, the following values being substituted for the constants of that equation: $a = +2{\cdot}0$ and $-1{\cdot}6$ for the two pole-pieces respectively; $b = 0{\cdot}04$. These pole-pieces are shown in section, in full line, in figure 4. As in the case of the pole-pieces described previously, the smaller one was elongated in order to reduce any effect of the outer part (*ab*) of the pole-face on the field in the gap. The force-distribution along the axis is shown by the curve of figure 5, and it will be seen that the theoretically uniform distribution is not too well realized.

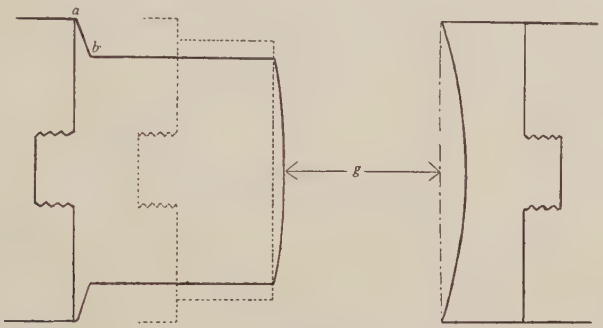


Fig. 4. Pole-pieces in section. Theoretically designed pole-pieces shown in full line; empirically corrected left hand pole-piece in dotted line.

Effect of annular pole-face. The calculations on which the design of the pole-pieces is based rest on the assumption that the field in the gap is due entirely to a distribution of magnetism over the faces of the pole-pieces. As, however, one pole-piece is of smaller diameter than the pole itself, there will be an annular area of the pole exposed, as at *ab* in figure 4. This will modify the field configuration in the gap, and in order to reduce this effect the smaller pole-piece was elongated, in the manner already described. In order that an idea of the magnitude of the effect of this part of the pole-face might be obtained, the following experiment was performed. A pair of pole-pieces was set up, consisting of a concave one of the full diameter of the pole-core and a plain cylindrical pole-piece of diameter 43 mm. and length 18·4 mm. at an available gap* of 34·8 mm. A force-distribution curve along the axis was taken, and the cylindrical pole-piece was then removed and faced off until its length was reduced to 15·3 mm. It was then replaced in the magnet, and the poles were brought together until the original gap was restored. A new force-distribution curve was then taken. The two curves are reproduced in figure 6 as (*a*) and (*b*) respectively. The apparent difference in magnitude of the force in the two cases is due to a change in sensitivity of the torsion balance caused by the disturbance due to the shifting of the poles on adjustment of the gap, but it is seen quite definitely that the shape of each curve is essentially the same. It therefore follows that the main geo-

* The term "available gap" is used as a convenient designation for the distance *g* of figure 4.

metrical factor deciding the shape of the force-distribution curves is the shape of the opposite faces of the pole-pieces, the approach of the annular shoulder by 3 mm. scarcely affecting matters. Further, with gaps of the order 30 mm., the contribution to the force in the gap, of the annular face of the pole at 18 mm. distance from the pole-piece face, is negligible. The distance 30 mm. was therefore standardized as the approximate thickness of the smaller pole-piece in succeeding experiments.

Effect of bored poles. The poles of the magnet were pierced with axial holes 11 mm. in diameter. In order to investigate any possible effect of this on the magnetization of the pole-pieces, and hence on the force-distribution curves, a test was made in which the bore was filled with a closely fitting soft-iron rod. Immediately on the completion of the observations of curve 6 (*b*), the bore behind the cylindrical pole-piece was filled with such a rod, and a curve was taken. The curve is shown as figure 6 (*c*), and is seen to be practically identical with figure 6 (*b*).

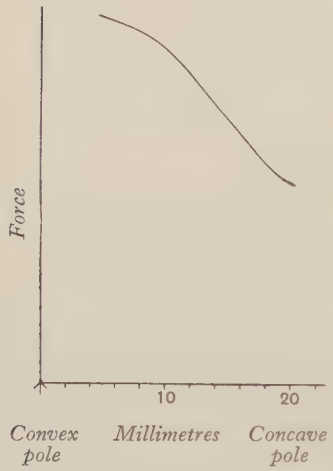


Fig. 5.

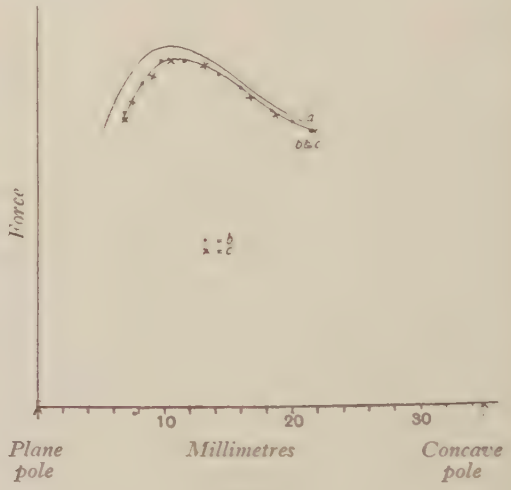


Fig. 6.

Hence the conclusion reached is that the nature of the force-distribution curve along the axis, for a given magnetizing current, depends only on the shape of the faces of the pole-pieces. The shape of the curves was shown to be independent of the magnetizing current also; the experiments are described later.

Improvement of field configuration. In the attempt to attain a more uniform force-distribution by modification of the pole-pieces, there are three principal ways in which an alteration can be made: (1) change of curvature of one or both pole-piece faces; (2) change of diameter of one or both pole-pieces; (3) change of distance between pole-pieces.

In order to simplify matters the concave pole-piece was left in position and all experiments were directed to modification of the convex one. A number of modified pole-pieces were tried, and it became apparent that the effect of a small change in any one of the above variables on the shape of the force-distribution curve could be almost exactly reproduced by a corresponding change in either of the others.

Thus a study of a progressive change in one variable gave force-distribution curves of all likely types. Such a study was made by observation of the force-distribution between the concave pole-piece and a series of plane pole-pieces, situated at a constant distance from it and having different diameters. Actually one pole-piece was used, and starting at the full diameter of the poles, it was turned to successively smaller diameters. The thickness of this pole-piece was 19 mm., and the available gap 33 mm. The force-distribution curves are reproduced in figure 7, and it will be seen that for one diameter of the pole-piece, namely 51·2 mm., curve (*e*), the force is uniform within close limits over a considerable region.

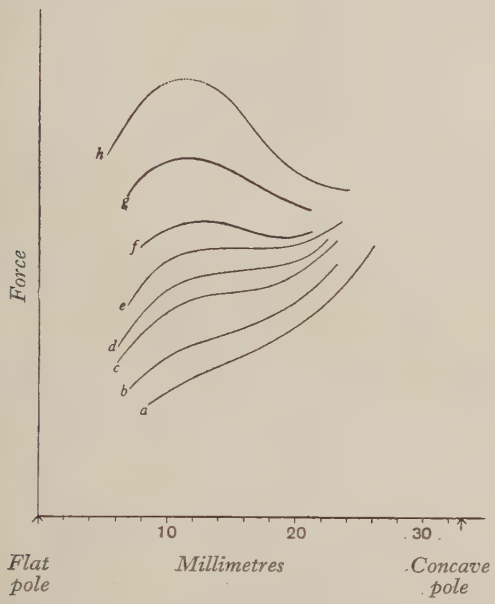


Fig. 7. Force-distribution along axis for the concave pole-piece of figure 4, and flat cylindrical pole-pieces of the following diameters: (*a*) 59·5 mm., (*b*) 57·1 mm., (*c*) 54·0 mm., (*d*) 52·4 mm., (*e*) 51·2 mm., (*f*) 49·6 mm., (*g*) 46·4 mm., (*h*) 43·0 mm.

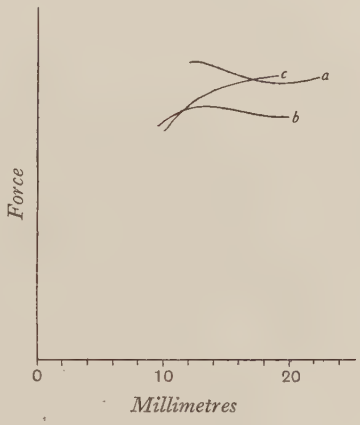


Fig. 8.

Change of curvature of pole-piece face. As has been mentioned above, it was found that a change of curvature of the pole-piece faces was equivalent to a change of diameter. To check this conclusion the face of the plane pole-piece was rounded off to a slight convexity in the case of the pole-pieces giving the distribution of curve 7 (*e*). The resulting force-distribution curve, figure 8 (*a*), is intermediate in shape between 7 (*f*) and 7 (*g*), so that the effect of the rounding is equivalent to a reduction of the diameter by about 2 mm.

Change of interpole gap. Curves (*c*) and (*b*) of figure 8 are force-distribution curves for the pole-pieces giving the distribution of figure 7 (*e*), but with the interpole gap slightly increased and decreased for the two curves respectively. It is again

seen that the effects are equivalent to those of a change in diameter; in this case there is also a change in magnitude of the force due to the change in field strength with the change in gap.

Variation with magnetizing current. As was explained in the previous paper, the pole-pieces were not magnetized to saturation. It therefore became of importance to determine whether the shape of the force-distribution curves depended on the degree of magnetization of the pole-pieces. A number of observations were made, and these showed that over wide ranges of magnetizing current the shape of the curves remained unaltered. Unfortunately, it was not possible, in the case of the magnet used, to reach saturation, but it is hoped to investigate the nature of the force-distribution at saturation on another magnet.

Variation of force perpendicular to the axis. For the pair of pole-pieces giving the most uniform distribution along the axis, observations were made of the force-distribution along a line perpendicular to the axis, at various points along the straight part of the curve 7 (*e*). The results, for distances of 9, 11, 13, 15, and 17 mm. respectively from the plane pole-piece face are shown in figure 9, from which it will be seen that the force at 2 mm. from the axis differs nowhere by more than 1 per cent. from its uniform value on the axis. It can be shown that the apparent asymmetry of some of the curves is due to a slight error in the adjustment of the direction of travel of the observing microscope.

More convergent field. According to theory, the field may be made more convergent, and the sensitivity of the method for susceptibility-measurements increased, if the curvature of the pole-pieces and the difference of their diameters be increased. A series of experiments in which a more concave pole-piece was used in conjunction with flat cylindrical pole-pieces of successively smaller diameters, was therefore carried out. As before, a pole-piece 19 mm. in thickness was used, and this was turned to smaller diameters; the available gap was again 33 mm. It had been observed that the pole-pieces designed according to the theory of the previous paper were almost exactly spherical in curvature, and for the purpose of these observations the concave pole-piece was therefore turned spherically concave, with a radius of curvation of 6 mm. The concave pole-piece of figure 4 is almost exactly a part of a sphere of radius 10 cm.

The force-distribution curves are shown in figure 10, and they are seen to resemble closely those of figure 7. It is obvious from a comparison with figure 7 (*f*) that curve 10 (*d*) corresponds to a pole-piece of slightly less than the optimum diameter.

Ageing. In some of the earlier experiments there seemed to be some evidence of an ageing effect, viz. a change with time in the shape of the force-distribution curve for a given pair of pole-pieces, and it was thought that a freshly machined pole-piece, if not annealed, might take some time to settle to a steady state. Some careful experiments indicated, however, that no appreciable ageing could be detected in a period of several days following machining.

§4. CONCLUSIONS

Although the pole-pieces designed in accordance with the theory give a force-distribution which is only approximately uniform, it is clear that a very slight modification of their shape is necessary to obtain a distribution which is uniform over a considerable region. The amount of modification necessary can be seen by reference to figure 4, in which the pole-pieces designed theoretically are shown in full lines, and those giving the nearly uniform distribution of curve 7 (*e*) in dotted lines. The observations indicate that the same result could be obtained by the use of the pole-pieces drawn in full lines, the interpole gap being slightly decreased.

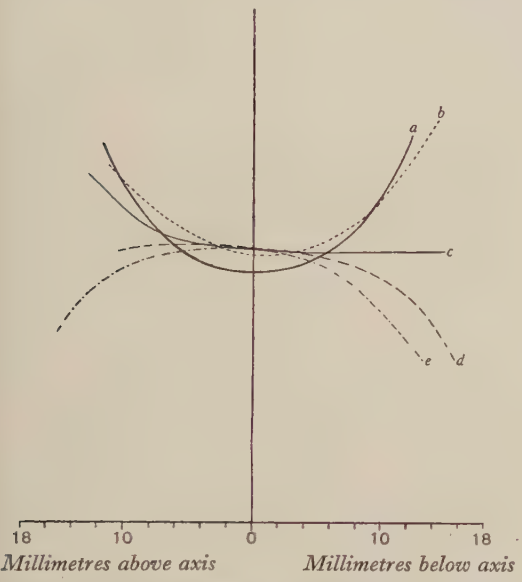


Fig. 9. Force-distribution perpendicular to the axis at various parts of the gap. Distances from plane pole-piece face: (*a*) 9 mm., (*b*) 11 mm., (*c*) 13 mm., (*d*) 15 mm., (*e*) 17 mm.

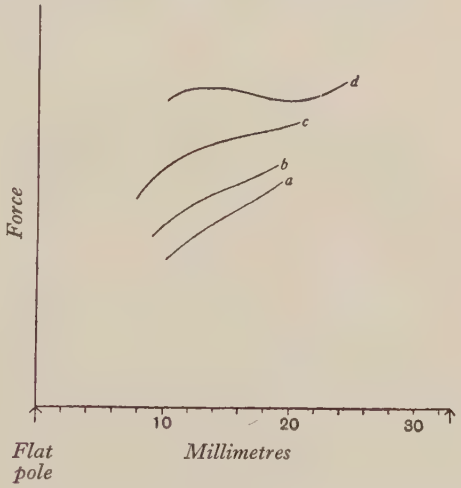


Fig. 10. Force-distribution along axis for more concave pole-piece and flat cylindrical pole-pieces of the following diameters: (*a*) 59·9 mm., (*b*) 57·1 mm., (*c*) 52·7 mm., (*d*) 49·4 mm.

The curves of figure 9 show that the force changes slowly and symmetrically as the point of observation passes away from the axis of symmetry. The field-configuration is therefore suitable for use in comparing the susceptibilities of moderately large specimens.

The concave pole-pieces designed in accordance with theory are practically indistinguishable from portions of spheres, and hence these force-distributions may be realized by means of the very simple pole-piece combination of a concave spherical pole-piece and a plane one. By the use of a more concave pole-piece, with the appropriate plane, the force on the specimen is increased somewhat, and the region of uniformity does not decrease in extent.

It is of interest to make a comparison between the new method and the more generally used Curie method for susceptibility-measurements. Curie* gives a curve showing the force-distribution $H_y \cdot \partial H_y / \partial x$ which shows two very sharply defined maxima, for forces in opposite directions. A similar curve is given by Owen† and is of the type reproduced in figure 11. It is seen that neither of the two maxima can be said to have a region of uniform force; the force increases to a sharp peak, on either side of which it falls off asymmetrically. The advantage of a force-distribution of the type of figure 7 (*e*) is obvious.

For the pole-pieces shown in full line in figure 4, theory indicates that

$$\partial H / \partial x = 0{\cdot}16\, H$$

at the centre of the gap. For the empirically corrected pole-pieces shown in dotted lines, the value found by experiment was about 0·10. With a magnetizing current of 5 amp. the field was about 3500 gauss, so that $H \cdot \partial H / \partial x$ was of the order 10^8.

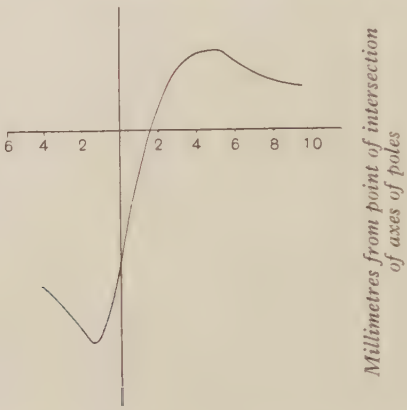


Fig. 11. Type of force-distribution curve obtained with inclined poles.

Some ratios of gradient to field-strength obtained by the Curie method are: 0·188‡, 0·145§, 0·43‖. Since the field-strengths are comparable in all cases, these ratios serve as a measure of the sensitivity of the apparatus. The value of 0·10 obtained in the case of the present apparatus, although below that obtainable by the Curie method, does not indicate a serious lack of sensitivity. A greater sensitivity has already been obtained, figure 10, but the value of the gradient to field-ratio in this case has not yet been determined. It would be possible to increase the sensitivity by working at the peak of a curve such as figure 7 (*h*). This would involve some sacrifice of force uniformity, but the uniformity attained would still be better than with a distribution of the type of figure 11.

It will be obvious that the new method should be particularly applicable to the determination of the principal susceptibilities of crystals, since a crystal could be set to the region of uniform force for the three successive measurements with a

* Curie, *Ann. Ch. Ph.* **7**, 5, 289 (1895), figure 1. † Owen, *Ann. d. Phys.* **37**, 657 (1912).
‡ Curie, *loc. cit.* § Hayes, *Phys. Rev.* **3**, 295 (1914). ‖ Owen, *loc. cit.*

reasonable certainty that each measurement would be made under the same conditions. Further, the method is inherently more suitable for comparisons of susceptibilities of magnetically anisotropic bodies than a method involving the use of a non-uniform field of *force*.

§ 5. ACKNOWLEDGMENTS

This work has been carried out in the Physical Laboratories at East London College, with the aid of a grant from H.M. Department of Scientific and Industrial Research. I have pleasure in expressing my thanks to Prof. C. H. Lees (at whose suggestion it was originally commenced) and to Prof. H. R. Robinson, for their advice and interest in the research, and for the facilities which they have placed at my disposal.

EDGE TONES

By E. G. RICHARDSON, B.A., Ph.D., D.Sc.

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ABSTRACT. If a fluid leaves an orifice as a jet and strikes an edge, two vortex streets are formed on each side of the wake, and maintain the jet in pendulation at a definite frequency. The tones so produced are examined from theoretical and experimental standpoints, and relations connecting the frequency with (*a*) velocity, (*b*) distance from orifice to edge, (*c*) width of orifice, (*d*) form of the orifice, are tested. It is shown that all the features of the phenomena can be explained in terms of the hydrodynamics of a viscous fluid, without postulating compressibility in addition.

§ 1. INTRODUCTION

When a fluid debouches from a circular orifice or slit into stagnant fluid a surface of discontinuity, as Helmholtz called it, is formed between the swiftly moving and the stationary fluid. Some of the latter is drawn into the jet and accelerated, with the result that the surfaces of discontinuity tend to spread out in the form of a conical or a plane surface as the fluid leaves the slit. The exact form of this spreading will depend upon the previous history of the jet, particularly upon the shape of the boundaries of the orifice and the method of approach. Generally there will be a vena contracta, just as in the case of a liquid projected into the air from an orifice; in which case the surface will not open out until the fluid has travelled a certain distance from the orifice. In this condition the jet is sensitive, for a very slight alteration in the pressure will cause a large change in the length of the jet as measured to the point where it opens out; this is the mechanism of the sensitive jet or flame. Experiments indicate that from the point where this mixing commences the mixing-zone increases linearly with distance along the jet, leading to plane or conical surfaces of discontinuity as the case may be.

Along these surfaces there are considerable shearing forces which give rise to the production of vortices. In the absence of periodic disturbance of the orifice itself, or of the formation of a stable vortex system, there is nothing to give rise to a tone in the jet. It is suggested that, in the production of an edge tone by the pushing of a sharp wedge into the stream of air issuing from a linear slit, disturbances of certain frequencies are encouraged and further stabilized by the eddies produced by friction on the solid wedge and may, with the vortices of opposite sense produced along the planes of discontinuity, give rise to two stable avenues of alternate and definitely spaced vortices of the Benard-Kármán type, one on each side of the wedge. These secondary eddies were noticed by Schmidtke* in photographs of the motion. The wave-length of this system determines the frequency of the edge-tone heard.

* *Ann. d. Phys.* **60**, 715 (1919).

§ 2. CALCULATION OF VELOCITY POTENTIAL AND OF FREQUENCY OF EDGE TONES

The axis of x being taken along the edge, and that of y through a pair of vortices (dotted line in figure 1), if h be the lateral and l the longitudinal spacing of each vortex-avenue, the complex velocity potential z at any point, due to the double street of vortices (assumed to be all of equal strength K), is to be calculated, where $z = x + iy$. x, y h, l z

The velocity-potential at z due to an isolated vortex at z_0 is w where w

$$w = (iK/2\pi) \log (z - z_0)^*.$$

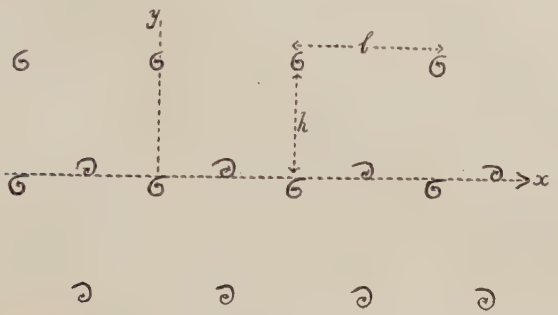


Fig. 1. Double street of vortices.

The corresponding expression for the two infinite series is given by

$$\begin{aligned} w &= u + iv \qquad u, v \\ &= \frac{iK}{2\pi}\left[\sum_{j=-\infty}^{j=+\infty} \log (z + jl - hi) + \sum_{j=-\infty}^{j=+\infty} \log (z + jl)\right. \\ &\qquad \left. - \sum_{j=-\infty}^{j=+\infty} \log \left(z + jl + \frac{l}{z}\right) - \sum_{j=-\infty}^{j=+\infty} \log \left(z + jl + \frac{l}{z} + hi\right)\right] \\ &= \frac{iK}{2\pi} \log \prod_{j=0}^{j=\infty} \left[1 - \left(\frac{z - hi}{jl}\right)^2\right]\left[1 - \left(\frac{z}{jl}\right)^2\right]\left[1 - \left(\frac{z}{jl - l/z}\right)^2\right]\left[1 - \left(\frac{z + hi}{jl - l/z}\right)^2\right] \\ &= \frac{iK}{2\pi} \log \frac{\sin\left\{\frac{\pi}{l}(z - hi)\right\} \sin \frac{\pi}{l} z}{\cos\left\{\frac{\pi}{l}(z + hi)\right\} \cos \frac{\pi}{l} z}. \end{aligned}$$

If z be taken at a point $(x, 0)$ along the dividing line of the two streets,

$$v = \frac{\cosh \frac{\pi h}{l}\left(1 + \cos \frac{2\pi}{l} x\right) - \sinh \frac{\pi h}{l} \sin \frac{2\pi}{l} x}{\cosh \frac{\pi h}{l}\left(1 + \cos \frac{2\pi}{l} x\right) + \sinh \frac{\pi h}{l} \sin \frac{2\pi}{l} x}$$

after real and imaginary parts have been separated and the expressions have been cleared of constant terms.

* See e.g. Lamb, *Hydrodynamics*, p. 665.

The above expression for the velocity perpendicular to the stream and imposed upon it by the two systems of vortices shows a simple harmonic variation of v with distance forward of the edge. It indicates, in fact, a stationary velocity-wave of which the maxima occur at distances l apart.

But the whole of the vortex system is moving with a velocity u from left to right; consequently a wave-motion will be imposed upon the jet as it emerges from the slit, and its wave-length will be equal to l, the longitudinal spacing of the vortex
n system. The frequency n of the edge tone will then be given by the simple formula

$$u = nl.$$

What may be imagined to happen is something like this: the fluid leaving the slit spreads out along two planes of discontinuity which pass the edge at a definite distance h from it. Along these surfaces of discontinuity vortices tend to form, while vortices rotating in the opposite sense tend to form along the edge itself. These roll up into two Benard-Kármán streets on each side of the edge with discrete vortices at a definite spacing of h, l, determined by considerations of stability. The stability of such a street in the neighbourhood of a semi-infinite wall has not yet been completely examined, but the analysis indicates that the system will tend to leave the wall and rapidly assume that value of h/l, namely 0·28, which is proper to a free Kármán system. The fluid subsequently issuing from the jet oscillates with a frequency determined by l and the velocity of the vortices. The usual formula given for edge-tone production is:

$$U/na = \text{a constant}.$$

u To fit this to the earlier formula we must assume (*a*) the velocity u of the vortex
U system to be a constant fraction of that of the jet U. (*b*) The longitudinal spacing,
l, a l, to be a constant fraction of the distance from slit to edge, a.

§ 3. EXPERIMENTAL INVESTIGATION OF VELOCITY-DISTRIBUTION

The main purpose of this paper is to examine how far these assumptions are justified. As there seemed to be no data on the distribution of velocity and turbulence in a jet emerging from a linear slit, an investigation of this question was first made.* Air was admitted from a reservoir under pressure into a wooden box measuring 40 × 20 × 16 cm. from which it emerged into the atmosphere through a vertical slit 6 cm. long, whose width could be adjusted by means of a micrometer head. This part of the apparatus was substantially the same as that used by Benton†. The jaws of the slit were thick and bevelled at 45° from inside. The velocity and amplitude of velocity-fluctuation at various points in the median plane through the slit were determined with a hot wire held vertically in a fork. The velocity-determinations were made from a knowledge of the steady change of resistance of the

* Tollmien has examined the spreading of jets theoretically and Benton traced out the surfaces of discontinuity by traversing the wedge across the slit till the edge tone ceased to be audible.

† *Proc. Phys. Soc.* **36**, 109 (1926). This paper contains a bibliography on edge tones.

wire, which had previously been calibrated for this purpose in a wind-channel. The amplitude of the velocity-fluctuations, taken as a measure of the vortex-strength, was found by coupling the hot wire through a transformer to a string galvanometer. The response of the latter, corrected for the average velocity in which the hot wire was placed, was taken as a measure of the vorticity, by the extent of the disturbance shown in the mean velocity. The pressure inside the box was measured with a Chattock gauge.

A characteristic plot of velocity and velocity-fluctuation is shown in figure 2 for a slit 0·09 cm. wide at a pressure of 8 mm. of water. It will be observed that the two surfaces spread from a vertex 1·2 mm. from the slit at an angle of 21°. For orifices wider than about 0·05 cm. this angle is changed but little, as the pressure is altered. Some values of this angle of spreading are shown in table 1.

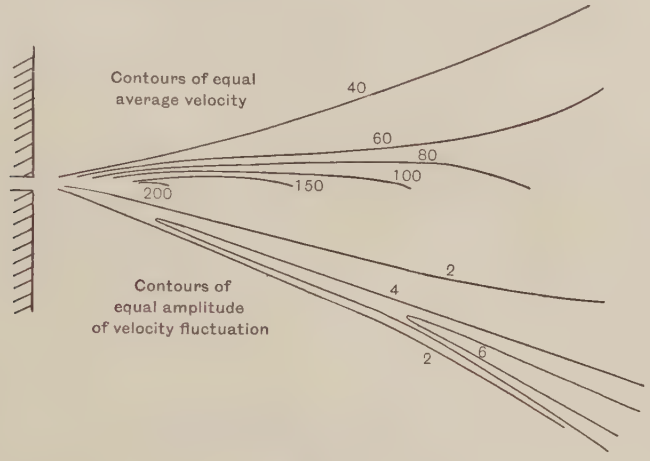


Fig. 2.

The results indicate that the angle θ of spread increases as the width d of the slit increases, reaching an ultimate value of approximately 20° for wide slits at high velocities of efflux. θ, d

Table 1

Width d of slit (mm.)	Pressure p (mm. of water)	Vertex at (mm.)	Angle θ (degrees)
0·9	0·65	10	10
	2·5	3·7	20
	8·0	1·2	21
	14·0	0	18
0·25	1·0	6·5	6
	2·5	3·2	12
0·5	1·0	7	16
	2·5	3·2	22

The next points to be tested experimentally are those involved in the two assumptions above: (1) the dependence of θ on the introduction of the wedge; (2) the

dependence of h/l on the introduction of the wedge. Figure 3 is a plot of velocity and velocity-fluctuation for the same jet as that to which figure 2 refers, with an edge of angle 15°, at 1 cm. from the slit. The secondary vortex-row is seen on each side of the edge arising from the tip. The streets broaden out somewhat after leaving the edge, besides being deflected. Figures 4 and 5, show the surfaces of discontinuity for wedges of angle 30° and 5° respectively. It will be observed that the

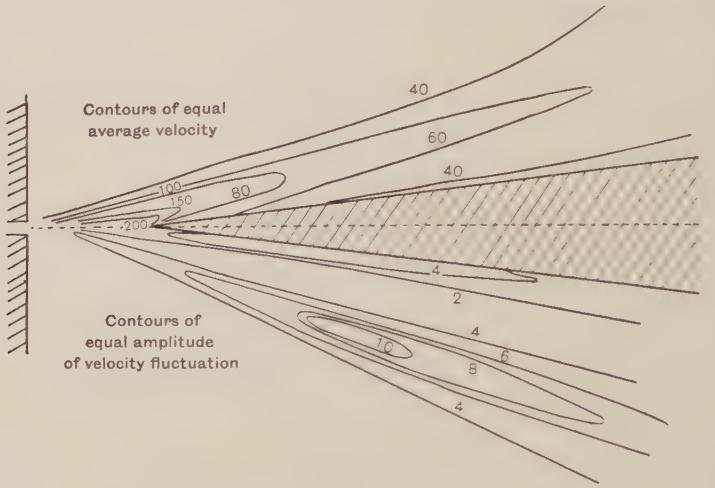


Fig. 3.

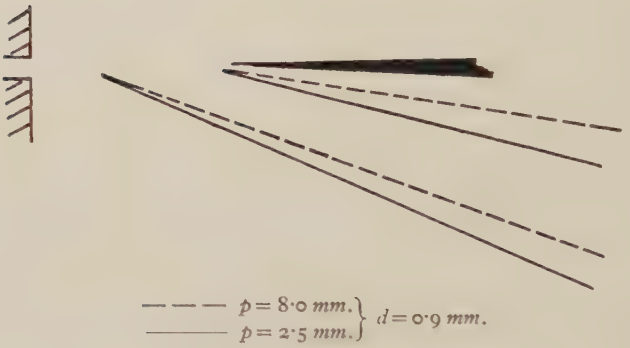


Fig. 4. Paths of vortices.

widths of the avenues are but slightly affected by the change of angle of wedge, but that they are deflected out from the tip to a greater extent as the angle of the wedge is increased.

That the vortex system is practically independent of the angle of the wedge is borne out by frequency-measurements. Stroboscopic examination of the response of the string galvanometer gave the values of frequency, for different wedges, shown in figures 6 and 7. The jump of pitch indicated by the short vertical lines should be noticed. With these wide-angle wedges the tones are more feeble, probably owing

to loss of energy in friction, as the jet has to turn a sharp corner. Blunting of the nose of the wedge has a similar effect. Weerth records no change of frequency when wedges of 15°, 47° and 138° are used.

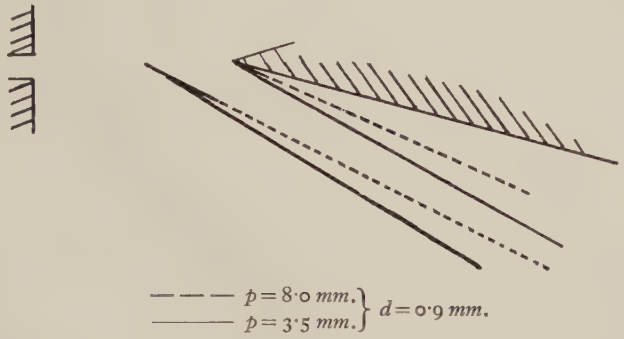


Fig. 5. Paths of vortices.

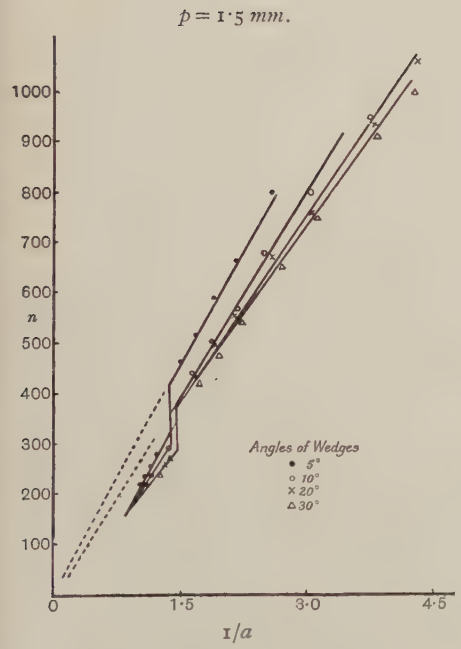


Fig. 6. Effect of angle of wedge.

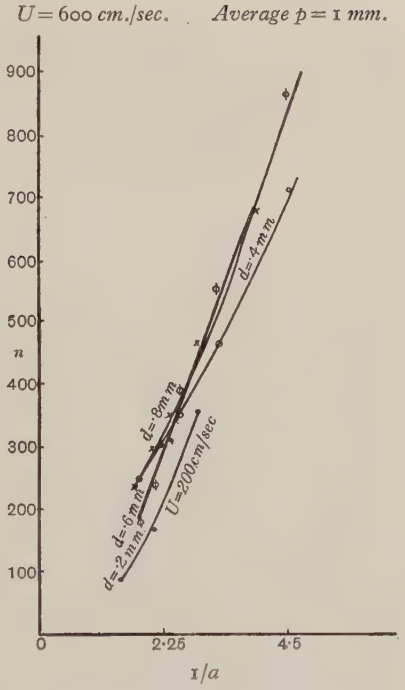


Fig. 7. Effect of width of slit.

The formula

$$U/na = \text{a constant}$$

depends on two assumptions: (*a*) equality between *a* and the "wave-length" of the vortex system; (*b*) proportionality between the velocity of the vortex system and the mean velocity of the jet.

The first of these assumptions may readily be verified by means of a photograph of the motion of a jet of coloured fluid emerging from an orifice and impinging on an edge. An apparatus for this purpose, designed by Dr R. S. Clay and kindly lent to me by him, is shown in figure 8. Water entering by the pipe on the right flows towards the left of the figure between two glass plates about 5 mm. apart. Ink drawn in from the series of capillary tubes serves to render the pendulation of the water visible as it leaves the slit and strikes the edge. Part of a cinematograph film of the motion is reproduced on the plate; the part of the apparatus which appears on the film is marked by the dotted line on figure 8. By following the series, a disturbance may be traced as it grows from the slit to the edge. In these pictures the wave-length of the disturbance is equal to the height of the mouth; in other cases photographed the wave-length was one-half or one-third of this distance. The secondary vortices coming from friction along the edge fall into line with the main vortices due to the pendulation of the jet, on either side of the edge.

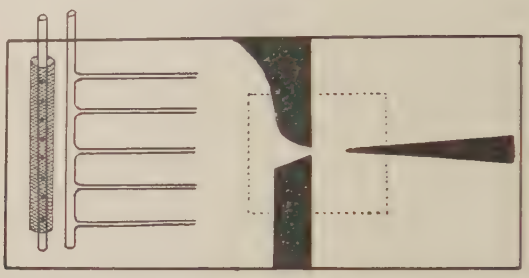


Fig. 8. Dr Clay's apparatus for demonstrating edge tones.

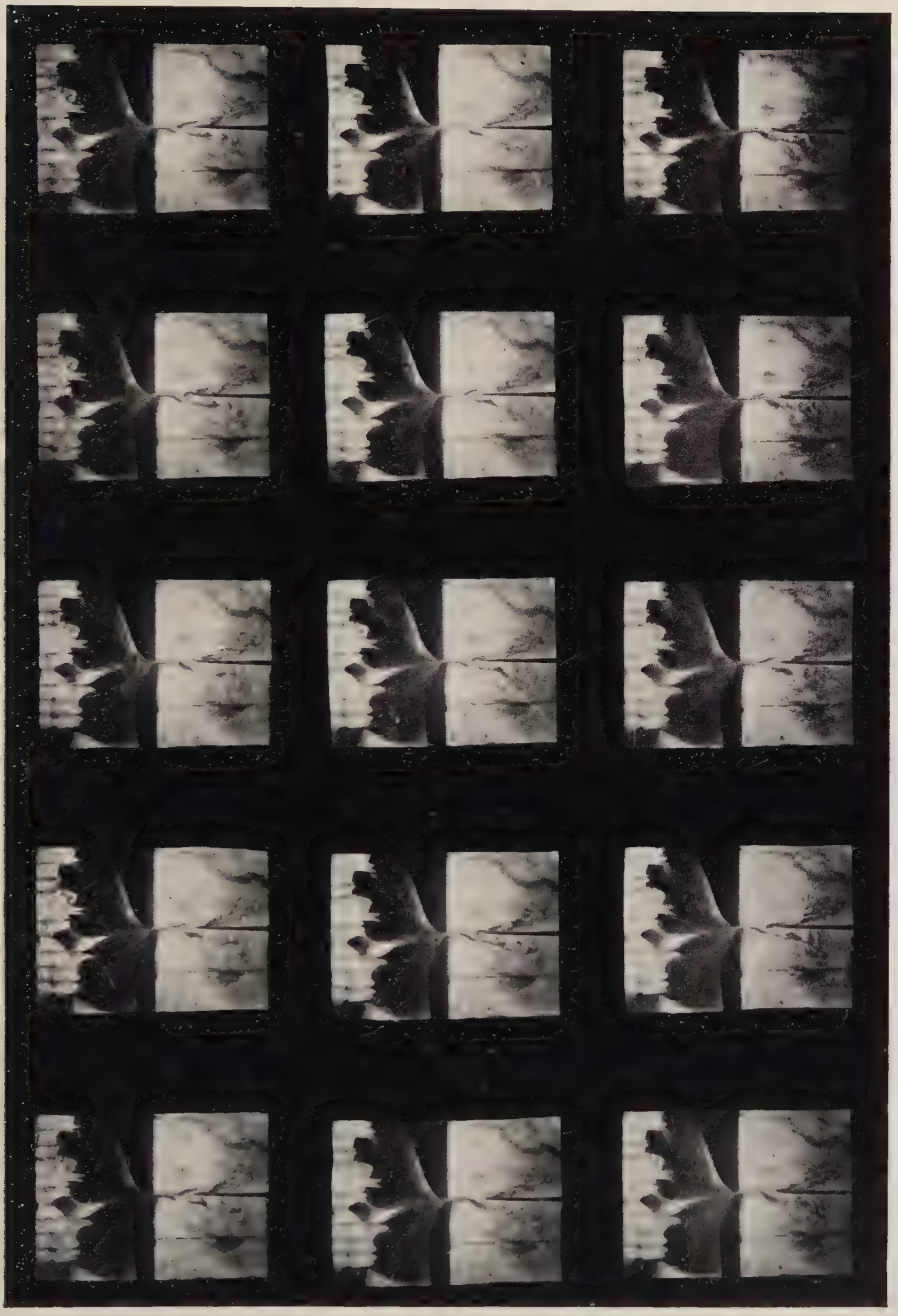
The wave-length of the vortex system can be measured by means also of a pair of hot wires, a method employed by Tyler* to measure the longitudinal spacing of the Benard-Kármán system of vortices behind a cylinder. For this purpose one hot wire was held in a fixed position by a bracket round the wedge, while the other was gripped by a traverse so that its fork could be run along the system longitudinally; further, each wire was capable of lateral adjustment on its fork. The current through each was led to a separate primary of a transformer, the two secondaries being then connected in series with the string galvanometer. The line of maximum velocity-fluctuation past the wedge was first fixed by the separate use of the wires, and the two wires were then placed in this path. As the movable wire was traversed, places of maximum deflection of the string galvanometer indicated that the disturbances were in phase, and therefore a wave-length apart. Results obtained by this method are given in the table 2. In most of the instances, the wave-length of the disturbance was half the distance from the slit to the edge.

With regard to the relationship between the velocity of the vortex system and the mean velocity of efflux from a linear slit, Rayleigh† has given a formula which is pertinent. For a jet of width $2b$ travelling with velocity U in stagnant fluid, the

b, U

* *J. Sci. Inst.* **6**, 310 (1929).

† *Scientific Papers*, 1, 368.



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relation between the velocity u of a disturbance of frequency n along the jet, so disposed that the sinuosities on either side are parallel, is given in the form u, n

$$(n + kU)^2 \tanh kb + n^2 = 0,$$

where k is the cycle-frequency $2\pi n/u$. k

The amplitude h of a wave of initial amplitude H travelling along the axis of x then becomes h, H

$$H \exp.(\pm \mu kUt).\cos k\,[Ut/(1 + \coth kb) - x],$$

where $\mu = \sqrt{(\coth kb)/(1 + \coth kb)}$. The wave-velocity u, viz. $U/(1 + \coth kb)$, tends to the limit $U/2$ for wide jets. The line on figure 9 shows the variation of u/U with kb. It is to be noted that in Raleigh's theory the frequency is quite arbitrary. μ, u

Table 2

p (mm.)	U (cm./sec.)	n	1 (cm.)*	a (cm.)
1·5	120	210	0·325	1·0
			0·325	
			0·30	
			0·29	
2·5	200	195	0·50	1·0
			0·49	
			0·47	
2·5	210	165	0·78	1·5
			0·74	
			0·70	

Since $u = nl$, we should expect the "constant" U/nl to be greater for narrow jets, more especially when the frequency is low. This is in fact the case, not only for edge tones of all kinds but also in the allied case of Aeolian tones, where the "constant" U/nd is greater for thin wires at low frequencies†, apparently from the same cause, for it makes no difference to the theory whether the stratum of fluid is moving rapidly past still fluid, or whether the main body of the fluid is in motion relative to the stagnant wake of an obstacle.

To test this formula of Rayleigh's against experiment, values of the frequency of the edge tone at various values of a, the distance of slit from edge, and of d, the slit width, were obtained and plotted on figure 10, the pressure being kept constant. The corresponding velocity of efflux U, which as a matter of fact varied little with the slit width, was obtained by means of a hot wire in the middle of the jet as it emerged. Having regard to the wave-length of the vortex system relative to a, the slopes of these lines in figure 10 together with those in figures 6 and 7 give the corresponding value of u, since $u = nl$. Points so obtained are shown as crosses on figure 9. a, d

It is to be noted that Rayleigh's theory takes no account of the dragging of stag-

* The numbers in this column give the distances between successive vortex pairs at any instant, passing outward from the slit.

† *Proc. Phys. Soc.* **36**, 163 (1924).

nant air by the jet and consequent spreading. When this spreading takes place in linear fashion from the orifice itself, as it does at higher pressures, l is an exact sub-multiple of a, and the edge-tone constant jU/na is given by the reciprocal of the ordinates in figure 9 (i.e. U/u), j being any integer.

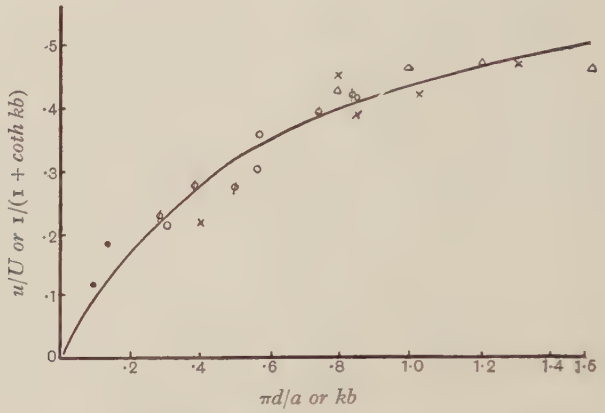

Fig. 9. Variation of wave-velocity with width of jet.

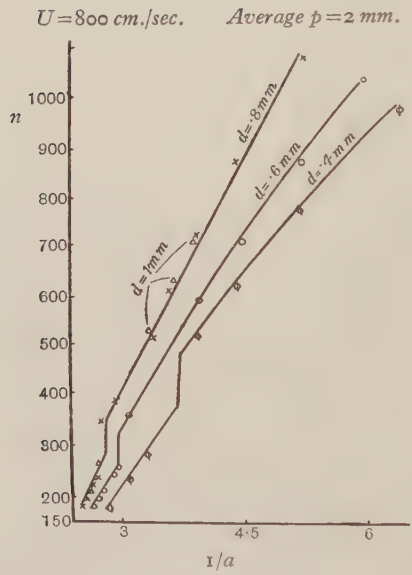

Fig. 10. Effect of width of orifice.

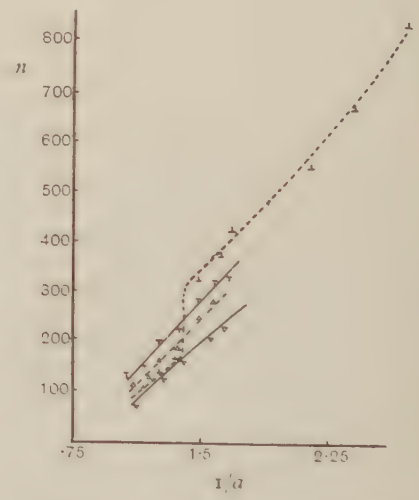

Fig. 11. Effect of thickness of jaws.
$p = 3$ mm., thin jaws ⊥, thick jaws ⊤.
$p = 2$ mm., thin jaws △, thick jaws ▽.

It is interesting to note in parenthesis that another formula given by Rayleigh for a cylindrical jet emerging at velocity U from a circular orifice of radius r gives us the wave-velocity corresponding to this case, so that

$$u = U/(1 + \eta^2)$$

where

$$\eta = \log 8/kr + \pi^{-\frac{1}{2}}\,\Gamma'(\tfrac{1}{2}).$$

The value of u is U, when $kr = 0$, and decreases as kr increases. The expression is valid only for small values of kr, so that the other extreme value cannot be deduced from it. This formula is applicable to the case of the bird-call where the jet emerges from a circular hole in a plate and impinges on another equal hole in a parallel plate a short distance away, and to the Galton whistle in which this second plate is replaced by a short pipe co-axial with the hole. If the orifice is partially closed by a circular disc we have the case of the annular jet, data for which have previously been obtained by the author and E. Tyler*.

§ 4. VORTICES FROM A HOLE IN A THICK PLATE

The system of vortex rings from a hole in a plate can be stabilized without the intervention of the edge if the plate be made thick; in this case the thickness of the plate acts as the determinant length in the edge-tone formula. The tones of such a system are, however, weak. As bearing upon the question of the influence of the spreading on the edge-tone wave-length, some measurements of frequency were made with the linear edge and slit, but with thick jaws to the slit, figure 11. The jet in this case spreads out less readily, and the velocity of the jet is reduced. Consequently the graph for the thick jaws lies below those for the thin ones at corresponding pressures.

§ 5. SUMMARY AND CONCLUSIONS

The fluid leaving a slit spreads out from a vertex a little beyond the orifice, dragging in stagnant fluid as it goes. Along the surfaces of discontinuity thus set up eddies are formed. Another set of secondary eddies leaves the tip of any edge inserted in the jet, and the two streets of discrete vortices so formed pass down the stream. The width of these at the tip of the edge is h or $a' \sin\theta$, where a' is the distance of the edge from the vertex and θ is the angle of spread. The ratio of the lateral spacing h to the longitudinal spacing l of the vortices in a Kármán street in a free stream is 0·28, so that $h = 0{\cdot}28\,l$. In practice $\theta = 20°$, whence $h = 0{\cdot}34a'$. Assuming that h/l is little changed by the proximity of the edge, and comparing the above formulae, we see that l will be nearly equal to a. The vortex system thus stabilized will produce a velocity potential along the issuing jet which will tend to make undulations of the same wave-length, which in turn will grow in amplitude as they pass along to the edge, where they will supply new vortices to the alternate procession, maintaining the original wave-length. a'

An alternative theory due to Krüger and examined by Schmidtke may be mentioned. In this theory, an embryo vortex leaving the slit and travelling with the stream starts off a new vortex from the slit when it hits the edge. A wave of compression then travels back with the velocity of sound to start the new disturbance from the jet. At the suggestion of Mr A. M. Cassie a crucial test of this theory was staged. In the apparatus of figure 8 an additional slit was made in the barrier, about

* *Phil. Mag.* **2**, 436 (1926).

5 cm. to the right of the existing one, but was not provided with an edge. The left-hand jet being stabilized to rhythmic pendulation by its edge, the wave of compression should have affected the right-hand jet equally. However, no definite periodic motion of this jet was noted at any velocity. This test, negative though it is, seems to dispose of the compression-wave theory.

§ 6. ACKNOWLEDGMENTS

In conclusion, I wish to express my best thanks to Prof. E. N. da C. Andrade, in whose laboratory this work was done, for his interest in the problem and for placing every facility at my disposal.

DISCUSSION

Dr L. Simons. The hiss of escaping steam from a safety valve is to my ear associated with a definite musical pitch which, I have noticed, apparently rises as I approach the source and remains constant if I am stationary. If I have not misjudged the phenomenon, is this associated with a regular vortex system as described by Dr Richardson? What would be the cause of the steady rise in pitch on my comparatively slow motion towards the source?

Author's reply. I can only suggest that the phenomenon described by Dr Simons is a reflection tone associated with the stationary-wave pattern between the listener and a neighbouring wall. The vortex system from a simple jet like the safety valve will have no edge to stabilize it; consequently the sound emitted is a mixture of frequencies, a high-pitched noise in fact, each component having nodes at different planes between the listener and the source. Movement towards the source will therefore cause a change in the quality of the note, which under some circumstances may sound like a rise in pitch. The same phenomenon may be observed at a large waterfall, owing to reflection from the rock behind, particularly if one can walk behind the waterfall towards the rock.

THE METHOD OF FORMATION OF SAND FIGURES ON A VIBRATING PLATE

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University of London, University College

Received May 5, 1931.

ABSTRACT. The conditions governing the movement of small particles on a vibrating surface are investigated experimentally and theoretically, close agreement being obtained. An arrangement by means of which the formation of the Chladni sand figures can be studied in detail, and projected on a screen, is described.

In connexion with a discourse delivered by one of us at the Royal Institution, an apparatus was constructed to enable Chladni figures to be projected epidiascopically. For this purpose a square steel plate, cut from a piece of Brown and Sharp ground stock, was mounted centrally on a pillar in the way usual for Chladni plates, but, to throw the plate into vibration, an electromagnet fed with alternating current was employed instead of the usual bow. The magnet *M*, taken from a loud-speaker unit, was fixed rigidly to the end of a pivoted arm, as shown in figure 1, the inclination of the arm being controlled by a screw *C* kept in contact with a

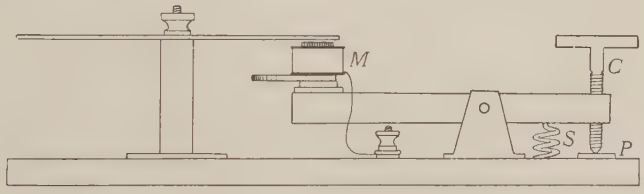

Fig. 1. Diagram of apparatus.

plate *P* by a spring *S*. The whole was mounted on a wooden board provided with levelling screws. Current to the magnet was provided by a valve-oscillator set, with suitable amplification, which gave a good output over a range of frequency from 100 to 8000 ~. The plate being levelled and strewn with sand, a range of beautiful figures is easily produced by progressive increase of the frequency, and careful tuning as soon as the plate shows signs of resonant response. With the help of an epidiascope the formation of the patterns of nodal lines, and the passage from one pattern to another, can be made visible to a large audience, which may be of interest to the many exponents of wave-mechanics who use the free modes of a vibrating plate as an analogy. This method of producing the figures is to be preferred over the older bowing method, not only because the vibration can be maintained at a fixed frequency and constant amplitude over a long period, but also because the vibration

is really free, which is not strictly the case with bowing, as was pointed out by Lord Rayleigh*.

This apparatus seemed to offer a good opportunity for investigating the question as to exactly why the sand moves to the nodes, which does not appear to have ever received serious attention. The only explicit reference which we have been able to trace is in Winkelmann†, who says: "Neither in the original publications nor in the textbooks is there any satisfactory statement as to how the sand figures are mechanically formed. For if we consider that a particular part of the plate bounded by nodal lines is alternately convex and concave upwards, and that the impulse of the particle of sand which is thrown upwards in one case has a component directed outward to the node, and in the other case inward to the loop, we must conclude that what occurs, on the average, is a hopping of the particle at a fixed and given spot: we must therefore consider the process more carefully." He then supposes that two cases can occur, either that the particle always falls on a surface convex upwards, or else alternately on a convex and concave surface, the third case (presumably that in which the particle always falls on a concave surface) not occurring, "weil die konkave Fläche allein keinen Impuls erteilt." He therefore states that the particle is always thrown outwards on the whole, since the first case moves it out, and the second leaves it stationary. While this appears plausible at first sight, as far as it goes, it is not really satisfactory, for it assumes implicitly that, in the second case, the particle is always thrown outwards from the convex surface by just the same amount (or at any rate not less) as it is thrown inwards when it falls on the concave surface. This is certainly not necessarily so. The point is examined at the end of the present paper. The process which leads to the particle leaving the plate is not indicated by Winkelmann, and will be now discussed.

The mechanism which suggests itself is as follows. As the plate moves down from the equilibrium position the acceleration is upwards, and the reaction between the particle and the plate is greater than the reaction at rest. This holds true while the plate is moving upwards until the equilibrium position is passed, but between this position and the extremity of the upward path the acceleration of the plate is downwards, and the reaction is diminished. If, before it reaches the highest point, the acceleration of the part of the plate considered becomes g, the particle will cease to be in contact with it, and will proceed upwards with an initial velocity equal to that of the plate at this moment. The subsequent path of the particle will be the parabola described by a projectile thrown upwards with this velocity, supposing that it be heavy enough for the turbulent motion of the air above the plate not to affect it.

If this is what actually happens, we should expect the sand to move on the plate until it reaches, not a nodal line, but a line at which the maximum acceleration of the plate is just g. Of course, if the motion of the plate is very vigorous, such lines

* "In certain cases deviations occur, which are usually attributed to irregularities in the plate. It must however be remembered that the vibrations excited by a bow are not, strictly speaking, free, and that their periods are therefore liable to certain modifications," etc. *Sound*, 1, 367, 368 (1894).

† *Handbuch der Physik*, 2, 385 (1909).

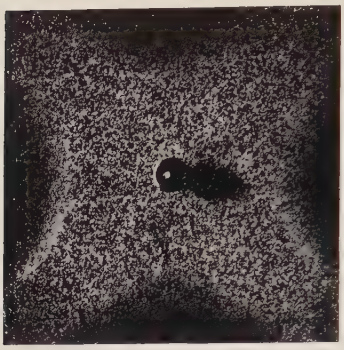

Fig. 2.

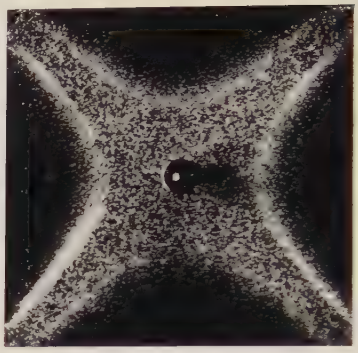

Fig. 3.

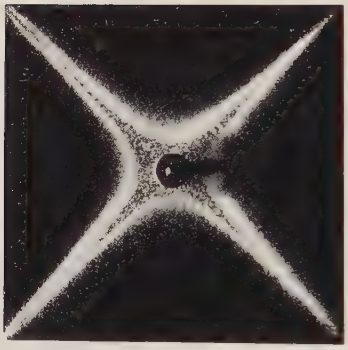

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

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will lie close to the nodal line on either side, but with a moderate motion of the plate they may be very far from it. This was easily shown to be the case with the apparatus of figure 1. Figures 2, 3, 4 and 5 show the appearance obtained with four different intensities of vibration. With a vibration just exceeding in energy that at which no movement of the sand takes place we have, after the plate has been left in steady vibration for some minutes, the state shown in figure 2: a stronger vibration, steadily maintained, leads to figure 3: figures 4 and 5 correspond to cases where the vibration is still more vigorous. The double lines can still be distinguished in figure 4; in figure 5 the sand is too thick for them to show separately.

To investigate the question quantitatively it is necessary to measure the maximum acceleration of the plate at the spot where the sand grains just do not move, that is, at the extreme boundary of the sand pattern. If the motion is represented by

$$s = a \sin nt,$$

this acceleration is n^2a. n can be ascertained by measurement of the frequency. To this end the magnet current was passed through a string galvanometer, and the motion of the string was recorded photographically on a film on which a standard time-marking was recorded also. The amplitude at a given place near the edge of the plate was taken as follows: a microscope is focussed on a bright spot on the edge, which is drawn out into a line when the plate is thrown into vibration, and the length of this line is measured with a calibrated eyepiece thread micrometer. With suitable illumination chance irregularities on the edge of the plate furnished bright points without any special preparation.

The first experiments were carried out with a square plate giving the diagonal cross figure shown in the plate. The sand used was washed successively with hydrochloric acid, water and ether and then strongly heated. The value for the maximum acceleration at the sand boundary came out to be 1480 cm./sec.^{2}. It was noticed, however, that a small chip of sealing-wax which happened to get on to the plate moved at a spot where the sand was at rest, and this led to the conclusion that sand is not a suitable substance for these measurements, as, apparently, it tends to stick to the plate. Particles were therefore prepared by crushing a piece of ordinary soda glass in a mortar, the fragments so prepared being of the order of half a millimetre across. With crushed glass the preliminary measurements on the square plate gave 1230 cm./sec.^{2}.

Our square plate was not very suitable for accurate measurement, as the frequency corresponding to the diagonal cross was 463 $\sim$, and consequently the amplitude was very small, of the order of a few μ, and difficult to measure with precision. For final measurements a long steel bar was therefore selected, whose frequency at the first overtone was 124·05 $\sim$. This was supported on knife edges, distant 0·224 of the length from the free end, so that the bar vibrated with a node at the mid-point; it was maintained in vibration by a magnet placed beneath, as with the square plate. Crushed glass was used as the particles. Figure 6 shows the appearance with a given intensity of vibration, the groups of particles about the three nodes having sharp boundaries: figure 7 shows the central node on an enlarged scale. Measure-

ments made with this bar gave astonishingly good results, as will be seen from the table, from which it appears that the mean acceleration is 978 cm. sec.$^{-2}$.

The amplitude given is the mean of ten readings in each case.

Table. Measurements with steel bar.

Frequency 124·05 ~.

Observation	Amplitude		Acceleration (cm./sec.^{2})
	(Microhead divisions)	(cm. × 10^{-3})	
1	66·8	1·629	990
2	65·0	1·585	962
3	66·5	1·622	984
4	65·8	1·604	974

As, when the plate is convex upward, there may be a slight tendency for the grain to slip outwards (although the tendency to slip inwards at the other extreme position would seem to counterbalance this), a rough calculation was made to see the magnitude of this possible effect. The coefficient of friction, measured roughly by tilting of the bar until the particles just slid on it, was taken as 0·5. It turned out that the particles will not slip until the acceleration of the plate has reduced the reaction of the particle to 0·3 M dynes, instead of the normal 981 M dynes, where M is the mass of the particle, or this effect in any case cannot exceed 0·03 per cent. of the main effect.

To see if sand really does show a sluggishness of motion as compared with glass particles an experiment was carried out with the washed sand. The value of the critical acceleration as measured with the bar came out to be 1608 cm. sec.$^{-2}$. Why the sand shows this sticking effect is not known to us. It may be that, in spite of the washing, it remains hygroscopic, and adheres to the bar by a film of moisture.

It is well known that lycopodium and other light powders do not move to the nodes, but form agitated heaps at the antinodes, and this was shown by Faraday to be due to the dust being carried by the turbulent motion of the air above the plate. It has often been stated that, *in vacuo*, lycopodium powder would behave in the same way as sand, but, as far as we can tell, the experiment has never been tried, nor is it remarkable, in view of the difficulty of maintaining a plate in vibration *in vacuo* before the valve technique was developed. The apparatus being to hand, it seemed worth while to test the point. The plate was placed in a large vacuum desiccator, the top of which was closed by a thick slab of plate glass, and the pressure was reduced by a Hyvac pump to a value of something less than 0·5 mm. of mercury, as judged by a discharge tube. At this pressure lycopodium powder gave a pattern very similar to that given by sand, as tested by the diagonal cross figure. It was noticeable, however, that it required a greater intensity of vibration to produce a given spacing with lycopodium than with sand. A rough measurement gave, as the critical acceleration, 4000 gm./sec.^{2}. Sand appeared to behave in exactly the same way *in vacuo* as in air.

We can now consider in a little more detail the movement of a particle on the plate. Let the displacement of a particular point in the surface of the plate be s where

$$s = a \sin nt,$$ s, a, n, t

and let s_g be the displacement at the time t_g when the acceleration downwards is g. s_g, t_g

Then
$$n^2 s_g = g \quad \text{and} \quad s_g \equiv g/n^2 \qquad \ldots\ldots(1),$$

while
$$\dot{s}_g = na \cos nt_g$$
$$= na\,(1 - g^2/n^4a^2)^{\frac{1}{2}},$$
$$t_g = n^{-1} \sin^{-1}(g/n^2a),$$

and hence the time t' measured from the moment when the particle leaves the plate is t'

$$t' = t - n^{-1} \sin^{-1} p$$

if we put
$$g/n^2a = p.$$ p

The position of the particle at any time t is therefore given by

$$x = g/n^2 + na\,(1 - p^2)^{\frac{1}{2}}\,(t - n^{-1} \sin^{-1} p) - \tfrac{1}{2} g\,(t - n^{-1} \sin^{-1} p)^2,$$

where x is the vertical displacement measured from the equilibrium position of the surface of the plate, or x

$$x/a = p + n\,(1 - p^2)^{\frac{1}{2}}\,(t - n^{-1} \sin^{-1} p) - \tfrac{1}{2}\,(g/a)\,(t - n^{-1} \sin^{-1} p)^2.$$

If the particle is to fall on the plate just when it is in its equilibrium position going down, then $t = \pi/n$, or coming up $t = 2\pi/n$, while if it is to strike the plate when at its lowest point $t = 3\pi/2n$, giving respectively as equations for p,

$$0 = p + (1 - p^2)^{\frac{1}{2}}\,(\pi - \sin^{-1} p) - \tfrac{1}{2} p\,(\pi - \sin^{-1} p)^2 \qquad \ldots\ldots(2),$$

$$0 = p + (1 - p^2)^{\frac{1}{2}}\,(2\pi - \sin^{-1} p) - \tfrac{1}{2} p\,(2\pi - \sin^{-1} p)^2 \qquad \ldots\ldots(3),$$

and
$$-1 = p + (1 - p^2)^{\frac{1}{2}}\,(3\pi/2 - \sin^{-1} p) - \tfrac{1}{2} p\,(3\pi/2 - \sin^{-1} p)^2 \qquad \ldots\ldots(4).$$

These transcendental equations can be easily solved by trial and error. For (2), (3) and (4) p is respectively 0·874, 0·340 and 0·586. The corresponding velocities of upward projection are, for (3) and (4), 0·940na and 0·810na. For the velocity v of the particle at a time ψ/n, we have v ψ

$$v = na\,\{(1 - p^2)^{\frac{1}{2}} - p\,(\psi - \sin^{-1} p)\},$$

which gives in case (3)
$$v = -\,1{\cdot}078na,$$

and in case (4)
$$v = -\,1{\cdot}584na.$$

Hence at a spot where $n^2a = g/0{\cdot}586 = 1{\cdot}706g$ the particle is thrown outwards at some angle ϕ with a velocity 0·810na, and strikes the plate when it is at its lowest point with a greater velocity, 1·584na, and at a greater angle ϕ/p or 1·706ϕ, if we suppose as a rough approximation that the particle is falling vertically and that the angle of the plate is proportional to the displacement. At such a spot then, the particle should be thrown inwards towards the node, and Winkelmann's general argument is quite misleading. ϕ

If the plate is vibrating vigorously the particle may drop on to it at its lowest point after the plate has executed $m + \frac{3}{4}$ complete vibrations, where m has any integer value. The solution for $m = 1$ is $p = 0{\cdot}202$. When p is small we may write, neglecting p^3 and higher powers,

$$-1 = p + (1-p)^{\frac{1}{2}}\left(\frac{4m+3}{2}\pi - \sin^{-1}p\right) - \tfrac{1}{2}p\left(\frac{4m+3}{2}\pi - \sin^{-1}p\right)^2$$

as
$$-1 = p + (1-\tfrac{1}{2}p)\left(\frac{4m+3}{2}\pi - p\right) - \tfrac{1}{2}p\left(\frac{4m+3}{2}\pi - p\right)^2,$$

or $$4p^2\{(4m+3)\pi + 1\} - p(4m+3)\pi\{(4m+3)\pi + 2\} + 4(4m+3)\pi + 8 = 0,$$

of which the solution is, to the required approximation,

$$p = \frac{4}{(4m+3)\pi}\left\{1 + \frac{16}{(4m+3)^2\pi^2}\right\}.$$

This gives for $m = 2$, $p = 0{\cdot}1173$,

and for $m = 3$, $p = 0{\cdot}0856$,

and therefore if $n^2a = 1{\cdot}706g$ or $4{\cdot}95g$ or $8{\cdot}53g$ or $11{\cdot}7g$, to take the first four values, the particle should be thrown towards the centre, in ascending order of vigour. Any plate which has a spot where n^2a has one of the higher values will, of course, have places where values of n^2a lower in the series occur. It is a remarkable fact which has not, apparently, hitherto been noticed, that when the plate is vibrating vigorously the clearance is not, as might be supposed, more complete than at feebler intensity, but on the other hand a large number of particles remain dancing in the neighbourhood of the places of most vigorous vibration as can, for example, be seen in figure 5. It is clear that, after a particle at a spot where n^2a has one of the values specified has been thrown towards a place of vigorous motion, it will soon reach a spot where it is thrown the other way, and since the particles are not in reality spherically symmetrical points, as has been tacitly assumed, it will probably eventually fall so as to bounce at a sharp angle, and be carried through the critical place and ultimately reach a place of rest where $n^2a < g$. While a particle at a place of vigorous vibration is thrown about until eventually it is carried through any difficult spots, a particle once arrived at a place of rest is withdrawn from circulation, so that the process pictured is plausible. When, however, there is a large number of regions, like contour lines on the plate, at which the particle is thrown inwards to the antinodes, it will clearly be more difficult for a trapped particle to escape to the nodal lines. This is why, when the plate is in very vigorous vibration, particles can be seen in lively motion near the antinodes.

To confirm the general correctness of this picture search was made for a place at which particles should be thrown outwards, towards the antinodes. For this purpose the bar already mentioned was set into vibration with an amplitude at the antinode somewhat greater than that corresponding to an acceleration of $1{\cdot}7g$, and

crushed glass was scattered over the surface. It was observed that there was a critical region, some 10·5 cm. from the antinode, such that the particles on the nodal side of it moved only towards the node. Particles in the critical region itself moved some towards the node, some towards the antinode. The amplitude at this critical region was measured microscopically. Five independent observations gave 113, 117, 115, 112, 112 as the amplitude in arbitrary scale divisions, the mean of which gave as the critical acceleration $1{\cdot}74g$. This agrees quite well with the calculated value $1{\cdot}706g$.

It was not possible to identify quantitatively the critical region where the acceleration has the value $4{\cdot}95g$ or one of the higher values, because the phenomenon is complicated by particles thrown outwards by the lower critical region so that the appearance is very confused.

Observations were made on a square Chladni plate also. The plate, strewn with sand, was thrown into vibration with a fairly high amplitude, corresponding roughly to figure 5. Sand particles at the outer edge of the sand figure were observed to be in motion, and some of them moved out towards the antinodal region and were finally thrown off the plate towards the middle of the edge. A small glass ball placed as nearly as possible on the nodal line showed no tendency to leave the nodal region, but if placed outside, and near the edge of, the figure it was set into motion, carried out towards the antinodal region, and finally thrown off at the edge of the plate. This is in accordance with the theory, when the vibration is so vigorous that there are many critical regions.

One of us (D. H. S.) is in receipt of a grant from the Department of Scientific and Industrial Research, and has much pleasure in acknowledging the help so rendered.

Note. After the measurements described were substantially completed, Dr R. E. Gibbs drew our attention to a brief note by Sir William Bragg on a chattering device for the measurement of small amplitudes*. After describing the instrument Sir William adds that chattering on a vibrating surface takes place if an^2 is greater than g, and says, "Illustrations can be found in the behaviour of objects on a Chladni plate," but no more.

* *J. Sci. Inst.* **6**, 196 (1929). See also A. B. Wood, *Sound*, 455 (1930).

STANDARDS OF MEASUREMENT, THEIR HISTORY AND DEVELOPMENT

By Sir RICHARD T. GLAZEBROOK, K.C.B., M.A., Sc.D., F.R.S.

The sixteenth Guthrie Lecture, delivered May 15, 1931.

§ 1. INTRODUCTION

When, some few months since, your secretary, Dr Griffiths, invited me in the name of the Council to deliver the Guthrie Lecture for 1931 and suggested I should speak on standards of measurement, dealing with their history and development, I accepted with a light heart. I welcomed the opportunity of helping to commemorate in this way the name of Guthrie. I remember him—few here can say that—when more than fifty years ago I began to take some share in the doings of the Society, then about five years old. When I was elected, Prof. W. G. Adams was president, to be followed in the next year by Lord Kelvin. Guthrie occupied the Chair five years after I became a member, and for the first ten years of its life the Society owed much to his fostering care, his zeal for its interests and his wise guidance. As I have said, I accepted your invitation lightheartedly; the subject proposed had always interested me. I thought I knew something about it and that the labour of preparation would not be overserious. And so I began to get ready, soon to be undeceived. My knowledge was but a small fraction of what there is to know. When does the history of weights and measures begin? May we start with the arm of King Henry I, A.D. 1120, the three barley corns which make an inch, or the dry grains of wheat taken from the middle of the ear by which the weight of the original silver penny was determined? or are we to go back to Roman and Greek times, or further still to the time of Solomon and the Temple or the days of Abraham, or even earlier to the days of Cheops the builder of the Great Pyramid? For there is no doubt that even before that, standards of measurement were used; the architect of the Pyramid laid out his plan with strict attention to his cubits and palms and digits.

And all this is very interesting—possibly more interesting, dare I say it, than the details of Sears's latest comparison of the yard and the metre, or Benoît, Fabry and Perot's determination of the number of wave-lengths of red cadmium light in a metre?

But what a task to describe all that has happened in the long story of weights and measures during the past 5000 years—for the date of the Great Pyramid is given by some authorities as about 2800 B.C.

§ 2. ANCIENT STANDARDS OF LENGTH

When man became a measuring animal he naturally took as his standards the parts of his own body; the cubit, the length of the forearm from the elbow joint to the tip of the middle finger, was his unit of measurement. The Egyptian hieroglyph for a cubit is a figure of the arm, and this unit was divided into spans, palms, digits, etc. There were various cubits, differing somewhat in length, and our knowledge of them has come to us in the main from Egyptian monuments. It appears probable, however, that the Egyptians obtained their cubit originally from Chaldaea; and one of the oldest standards we know of is to be found in the Louvre, where there is a bust of Gudea, a Sumerian King of about 2300 B.C. On it there is engraved a scale showing sixteen digits of the Sumerian cubit, which was divided into thirty digits. Now this scale is 264·4 mm. long, so that the cubit of thirty digits is 496 mm. or 19·6 in. in length, giving as the length of the digit 0·653 in.*.

The cubit, from which it appears probable that our foot and inch have descended, is generally known as the Olympic cubit, because it passed from Egypt into Greece.

It was, as we shall see shortly, about 18·24 in. long and was divided into two spans, the stretch from the thumb to the little finger, of about 9 in. A span was equal to three palms, the breadth of four fingers, each being about 3 in. in length, and the palm was divided into four digits, each $\frac{3}{4}$ in. approximately, or, more accurately, 0·76 in.

There was also the fathom of 4 cubits 72·96 in. or 6·08 ft., the outstretch of the two arms.

At a somewhat later date another unit, the foot, equal to two-thirds of a cubit, was introduced, and this was divided into twelve thumb-nail breadths—Greek *ονυξ*—whence came the terms uncia, ounce, inch, used for the twelfth part of the Troy pound and of the foot.

I have not been able to hear of an existing specimen of an Olympic cubit, but we can determine its length in the following manner. Herodotus (*Euterpe*, 168) tells us that "the Egyptian cubit is equal to that of Samos," i.e. the cubit used in Greece. We can obtain the length of this from the measurement of various Greek temples.

The Hecatompedon of the Parthenon is a square of 100 ft. in side. This was measured in 1846 by F. C. Penrose†, who gave as the length of the Greek foot 12·1610 in., whence the cubit being 3/2 foot is 18·2415 in.; this was corrected later to 18·2405, or (say) 18·24 in.‡.

* I owe this information and indeed much of what I have gathered about the Egyptian measures and their relations to our measures to the extreme kindness of Col. N. T. Belaiew, C.B., the distinguished metallurgist who has made a long study of these matters and most generously put his knowledge at my disposal. Mr Sidney Smith, of the British Museum, has most kindly replied to various queries.

† F. C. Penrose, afterwards the distinguished architect and Fellow of the Royal Society, was sent out to Greece by the Society of Dilettanti to test certain theories on Greek architecture by measuring the buildings. As an undergraduate he rowed in the Cambridge boat in 1841, 1842 and 1843.

‡ Notes by Col. Sir H. James, Director General Ordnance Survey, 1869.

There is another link to be obtained thus. The side of the Great Pyramid is stated to be 500 cubits in length. This has been frequently measured. Sir Henry Lyons informs me that the result obtained a few years since by Mr J. H. Cole, of the Egyptian Survey, was 230·364 metres or 755·76 ft. Earlier measures give much the same result. Sir Flinders Petrie* found the average length to be 9068·8 in. or 755·7 ft., giving for the length of the cubit, if we accept the statement that a side of the base of the Pyramid measured 500 cubits, the value 18·14 in.

This is nearly the same as the 18·24 in. found from the Greek temple, and we may perhaps accept this value as close to the truth.

This foot with its division into twelve thumb breadths (ὄνυξ) became the Greek foot.

In due course the foot passed to Rome, becoming somewhat altered in value; the Roman mile of 5000 Roman feet is generally said to be equal to 8 Greek stadia each of 600 ft. or 400 cubits, so that the Roman foot was equal to 24 25 of the Greek foot or 11·67 English inches.

This makes the Roman foot equal to 0·973 English feet. Direct measurement by Martin Folkes† and others give as the value 0·967 English feet; or, putting it the other way round, 1 English foot = 1·03 Roman feet.

In time Roman measures were established in Britain, but as to when and how the change from the Roman to the British foot took place we have no evidence. Various surmises are possible. I refer to one more fully on p. 421. For another it is known that the Gothic nations in the 4th and 5th centuries, established in the neighbourhood of the Crimea, used a large cubit. This became the basis of the Russian measures prior to the reign of Peter the Great, and may have travelled westward with the Goths and been brought to England by the Anglo-Saxons‡.

But more likely it was a chance alteration. Maybe King Henry I, A.D. 1100, did fix the yard by the outstretch of his arm and this happened to give a foot some 3 per cent. in excess of the Roman foot. There was a standard foot fixed in old St Paul's, and all measures were referred to the standard, "qui insculpitur super basem columpnae in Ecclesia Sancti Pauli."

King David of Scotland§ (*c.* 1150) is said to have ordained that the Scotch inch should be the mean measure of the thumbs of three men, "an merkle man an man of measurable stature and an lytell man," the thumbs being measured at the root of the nail.

According to Mr Nicolson, the earliest table of English linear measures is probably that in Arnold's *Customs of London* (*c.* 1500).

* Flinders Petrie, *Pyramids of Gizeh*, p. 39. For a further discussion of the relation between the cubit and the dimensions of the Pyramid, see p. 419.

† An account of standard measures in the Capitol at Rome by Martin Folkes, V.P., *Phil. Trans.* (1736). The ratio had also been determined by John Greaves, who in a visit to Italy in 1637 measured various standards, giving as the results the values 0·967, 0·972 and 0·986. For fuller details see p. 417.

‡ This suggestion has been made by Col. Belaiew in his writings on Russian measures (*The History and Development of Russian Weights and Measures*).

§ For this and much other information relating to early measures I am indebted to *Men and Measures* by Edward Nicolson.

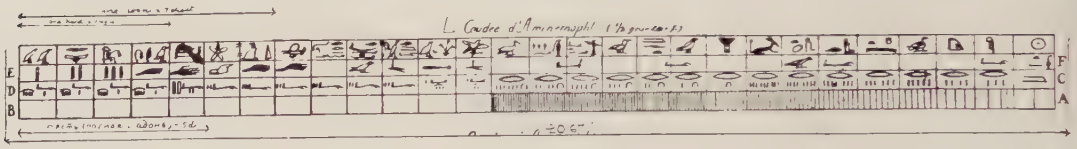


Fig. 2. The Royal Egyptian cubit = 2 half-cubits = 4 spans (of 7 digits) = 7 hands (of 4 digits).

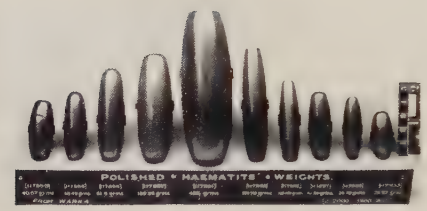


Fig. 1. Haematite weights from Warka (Erech). British Museum.

Reproduced with the kind permission of the Director and Trustees of the British Museum.

Fig. 6.

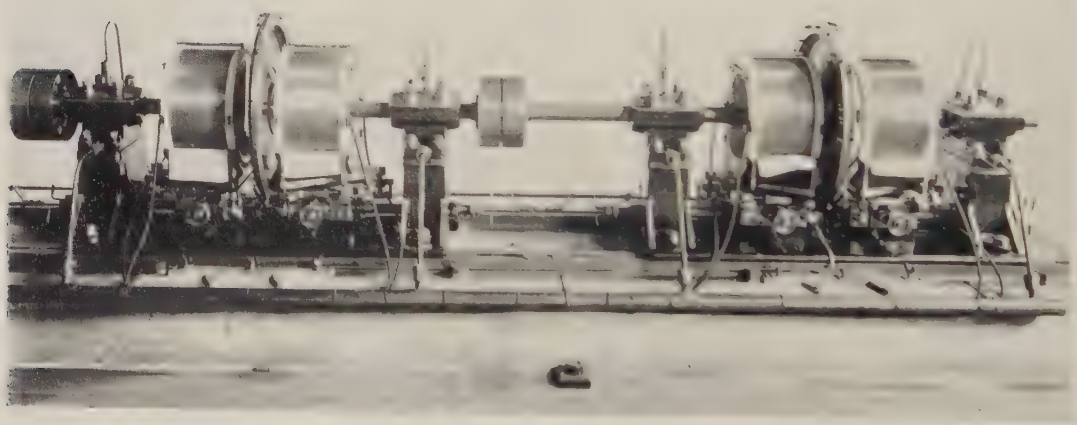


Fig. 5.

"The length of a barly corne iij tymes make an ynche
and xij ynches make a fote
and iij fote make a yerde
and v quarters of the yarde make an elle
v fote make a pace
cxxv pace make a furlong
and viij furlong make an English myle."

Thus in A.D. 1500 the furlong was 625 ft. or 208⅓ yds. and the mile 5000 ft. or 1666·6 yds. Half a century later, in the time of Queen Elizabeth, steps were taken to fix definite standards of measurement, but before I tell of these it is desirable to refer to the history of weights.

§ 3. ANCIENT STANDARDS OF WEIGHT

The standard of weight in Sumerian times was the mina, the history of which has been very fully examined by Col. Belaiew*.

Figure 1 is an illustration of a series of these mina weights made of homatite now in the British Museum. According to his investigations there were three series of standards, one of which became specially associated with the weights of coins. Another may have relation to the weight of water in a cubic cubit. From the mina in course of time came the talent, and the Alexandrian talent formed the basis of the Greek, Roman, and Western European weights until the advent of the Metric System.

The Alexandrian talent was divided by the Romans into 125 librae, each containing 12 unciae, so that there were 1500 oz. to the talent. The weight of the talent has been found to be 93·65 lb. avoirdupois or 655,550 grains. Thus the weight of the Roman ounce was 437 grains, while the pound contained 5244 grains. This weight became current in Britain, but the pound, instead of remaining at 12 oz., was raised to 16 oz. each of 437 grains and possibly became the basis of our avoirdupois weight, a pound of 6992 grains. Another suggestion is that the pound avoirdupois came from the Attic mina of 6945 grains.

The Saxon kings, however, had brought to England as a standard weight the marc of Cologne. Two marcs weighed 7216 grains and were divided into 16 oz. each of 451 grains. Twelve of these ounces, reckoned as weighing 450 grains each, constituted the Tower or Mint pound by which the coinage was weighed. In the time of Edward III (about 1350) a Tower pound of silver was coined into 240 silver pennies each weighing 22·5 grains, giving 5400 grains† for the pound and 450 for the ounce.

But there was another pound in ordinary use, the Troy pound. Its origin is quite uncertain. In France we find a *marc de Troyes* of 472 grains to the ounce, and it may have come from that. Another suggestion is that the word Troy has no reference to the French town but rather to Troy Novant, a monkish name of London

* "On the Sumerian mina, its origin and probable value," *Trans. Newcomen Soc.* 8–(1927–8).

† In 1842 a Tower pound from the Pyx Chamber was found to weigh 5404 grains.

founded on the legend of Brutus. Troy weight was mentioned in the reign of Henry IV, and in the time of Henry V goldsmiths were ordered to use "la libre de Troy" and coin 360 pennies from a pound of 5400 grains. Thus a penny weighed 22·5 grains, and an ounce 450 grains.

The Troy pound did not become the standard for the coinage until 1527, when Henry VIII enacted that "the pounde Towre shall be no more used, but all manner of golde and sylver shall be wayed by the pounde Troye which maketh XII oz Troye and which excedith the pounde Towre by III quarters of the ounce." This would make the pound Troy weigh 5400 – $\frac{3}{4}$ (450) grains or 5737·5 grains. It was afterwards defined as 5760 grains and the ounce Troy (1/12 part) as 480 grains.

But throughout this time avoirdupois weights, though not defined by statute until 1485, had been used, and from quite early days a *libra mercatoria* one-fourth greater than the Tower pound (containing therefore 6750 grains) was employed.

In the time of Edward III (1352) we read of weighings "cum ponderibus de haberdepase" and weights of this denomination existed in the time of Elizabeth. Specimens are on the table. "Haberty poie" pound is mentioned in the time of Henry VII. Henry VIII (1532) ordered that meat shall be sold by weight called haver-du-pois*. The avoirdupois pound contained 16 Roman ounces of 437 grains each—6992 to the pound.

Elizabeth, by her action in 1584, introduced what has become our modern system of weights and measures.

She ordered a jury in 1574 to examine the standard weights and measures in use, to report on them, and to construct standards "as well of Troy weight as of avoirdupois." The standards were found to be very erroneous and a second jury was appointed in 1584. The result of their work was the production of 57 sets of weights, both Troy and avoirdupois, which were distributed to the Exchequer and various local authorities. Some of these weights are still extant and do not differ by more than a grain from the Imperial standard.

Either at this period or previously the weight of the avoirdupois pound was raised by 8 grains, being brought from 6992 to 7000 grains at which it has remained ever since. Thus the avoirdupois ounce is 437·5 grains.

The Proclamation for Weights of December 16, 1587, established avoirdupois weight and ordered that no person shall use any Troy weight but only for weighing of bread, gold, silver and electuaries and for no other thing†.

In addition standards of length and capacity were constructed; the standard yard of Elizabeth, now on the table, is continually referred to later and was an important factor in determining the length of our present standard.

The work was well done, and supplied in a satisfactory manner the needs of the country for more than 150 years.

* According to the *New English Dictionary* quoted by Nicolson, to whose book I am indebted for much of this account of English weights, "averdepois" is the best spelling of the name. "Aver" is an old established English word for "goods."

† Nicolson, *Men and Measures*, p. 102.

§ 4. THE PYRAMIDS AND THE CUBITS. EGYPTIAN STANDARDS

But before continuing the history of these standards it is of interest to go back to Egyptian times and to the Pyramids and learn more about those standards from which ours have undoubtedly come. I have based our foot on the common or Olympic cubit of 18·24 in., but there were many other cubits. One in particular, the royal cubit of Egypt, is closely connected with the building of the Great Pyramid.

John Greaves, Savillian Professor of Astronomy at Oxford, A.D. 1648–9, became Professor of Geometry in Gresham College, London, in 1631, and after some visits to the Continent started on a journey to the East in 1637. After visiting Rome and Constantinople he arrived at Alexandria in 1638 and thence "went twice to Grand Cairo to measure the Pyramids, carrying with him a Radius of ten foot most accurately divided into 10,000 parts besides some other instruments for the fuller discovery of the truth*."

Greaves was an Arabic scholar as well as a mathematician and astronomer and was the author of works on many subjects. One deals with the Grand Seignior's Seraglio or the Turkish Emperor's Court.

In the *Dissertation on Cubits* Newton sets down the dimensions given by Greaves for various parts of the Pyramid. He then argues that the builder probably used some unit† in the design of his work and, by comparing the figures, endeavours to fix on that unit; the result is shown in table 1, in which are included the dimensions of two of the principal passages and of the great central chamber as well as the length of one side of the Great Pyramid.

Table 1. Greaves's measurements of the Pyramids.

Part measured	Dimensions (ft.)	Value of cubit deduced
Breadth of entrance gallery	3·463	2 × 1·732
Breadth of main gallery	6·870	4 × 1·718
Height of side benches in main gallery	1·717	1·717
Breadth of side benches in main gallery	1·717	1·717
Length of central chamber	34·380	20 × 1·719
Breadth of central chamber	17·190	10 × 1·719
Length of north side of base	693	400 × 1·732

* "The life of Mr John Greaves," by Thos. Birch, Sec. R.S., prefixed to the *Miscellaneous Works of Mr John Greaves, Professor of Astronomy in the University of Oxford*. Published by Thos. Birch, F.R.S., 1737.

The works include among other writings: "I. Pyramidographia—a Description of the Pyramids of Egypt." "II. A Discourse of the Roman Foot and Denarius," and to them is added: "A Dissertation upon the Sacred Cubit of the Jews and the Cubits of the Several Nations; in which, from the Dimensions of the greatest Egyptian Pyramid, as taken by Mr Greaves, the antient Cubit of Memphis is determined." Translated from the Latin of Sir Isaac Newton. Not yet published.

† The method had been used previously by Greaves himself who, to check the value of the Roman foot as deduced from one of the monuments examined, measured the stones in the foundations of various buildings, especially those of the Pantheon, and found "that most of the white marble stones in the pavement contained exactly three of those Roman feet on St Cossutius' monument, and the lesser stones in porphyry contained one and a half." Greaves's *Dissertation of the Roman Foot*, p. 211.

The mean value for the cubit, excluding the first and last, Newton takes as 1·719 ft. The last line of the table, as will appear shortly, is clearly erroneous; as to the difference between the first line and the others Newton remarks that it comes to about 1/7 in., which is "an error of no importance if we consider the much greater irregularities observed by Mr Greaves in the best buildings of the Romans."

As to the base of the Pyramid, Greaves writes: "For, measuring the North side of it at the basis by an exquisite radius of ten feet in length, taking two several stations as Mathematicians use to do when any obstacle hinders their approach, I found it to be six hundred and ninety three feet according to the English Standard."

This length we know to be wrong. Until recently there was much débris about the base of the Pyramid and it was impossible to approach the actual corners. Newton, however, finding that the length of the cubit as determined by the other measurements was about 1/400th of the base, and knowing also that an Arabic writer, Ibn Abd Alhokm, quoted by Greaves, had stated that there were 100 royal cubits in the base, assumed that by "royal cubits" were meant the *orgyia* of the Greeks—four cubits in length—and that the cubit used was therefore 1 400 of the length of the base of 1·732 ft.

If we take Newton's value 1·719 ft. as the outcome of Greaves's measurements in 1638–9, we obtain, as the length of the royal cubit used in the interior of the Great Pyramid, 20·628 in.

There is ample evidence that a cubit of about this size was in general use. There is one in the British Museum—a model of this is on the table—another in the Liverpool Museum and a third in Paris*. Mr Nicolson mentions others.

The Paris cubit, figure 2, contains many interesting features. Its length is given as 20·67 in. It is divided into 28 digits†, each of 0·736 in. in length. Of these 15, measured from the right, are subdivided respectively into 2, 3...16 parts. The number of the divisions is indicated above line *CD* by lines thus: |||, |||, etc., ∩, ∩|, the ∩ stands for ten. On the top line are a series of hieroglyphics presumably indicating the number of digits. Reckoning from the left the digits are shown separately. Four of them constitute a palm shown as such—the royal cubit had seven palms—while five digits make a hand.

The royal cubit is found also on the outside of the Great Pyramid; at each corner were vertical walls, used by the builder to set out the slope of the faces. Two of these are extant and are marked in cubits of this size. From this it would appear that the royal cubit was used throughout the building.

Moreover, there still exist a number of Nilometers used in ancient times to measure the height of the Nile flood along the valley. These are marked in cubits. Several of them were measured by Sir Henry Lyons and Dr Borchardt when in Egypt, with the following results:

* Through the kindness of Col. Belaiew I am able to exhibit a copy of this cubit, while Prof. Wilberforce has very kindly measured the Liverpool cubit for me, with the result given in Appendix 2.

† The Olympic cubit of 18·24 in. was divided into six palms, each of 3·08 in. in length, so that the length of its digit was 0·77 in.

Nilometer	Number of cubit lengths	Average cubit (metres)
Philae I	12	0·520
Philae II	12	0·532
Philae III	16	0·535
Elephantine	12	0·523
Edfu	6	0·528
Esna	9	0·532
Luxor	6	0·529

The mean of these is 0·528 m. or 20·8 in. Other measurements have given a rather less result.

The mean value found by Mr Cole for the length of the side of the base was 755·8 ft. If this is to contain an even number of cubits of about the length found otherwise for the royal cubit, that number will be 440. This being assumed as the number of cubits in a side, the length of the cubit is 1·718 ft. or 20·62 in.

Table 2 gives the lengths of the four sides of the base as measured by Sir Flinders Petrie and Mr J. H. Cole.

Table 2. Base of the Great Pyramid.

	Sir Flinders Petrie	Mr J. H. Cole
North	755′ 9″	755′ 5″
East	755′ 8″	756′ 1″
South	755′ 9″	755′ 11″
West	755′ 9″	755′ 10″
Mean	755′ 8″·7	755′ 10″

The agreement is practically complete.

Thus we have found good cause to suppose that there were two cubits in general use—there were others besides—the one 18·24 in. in length, the other 20·62 in. It is perhaps natural to enquire what was the reason for this, and Col. Belaiew pointed out to me a fact of great interest in this connexion.

If we call l and L the lengths of the two, we have

$$l/L = 18{\cdot}24/20{\cdot}62 = 0{\cdot}885 = \tfrac{1}{2}\sqrt{\pi} \quad \text{approximately.}$$

Thus $$l^2 = \tfrac{1}{4}\pi L^2.$$

Thus the area of a square having the smaller cubit for a side is equal to that of a circle with the large cubit for its diameter*. This may, of course, be chance or it may be that some ancient Egyptian, wishing to construct a circle of the same area as the square, found the large cubit. We know that the value 256/81 was used for π, so that $\sqrt{\pi} = 16/9$.

But there are other curious facts connected with the dimensions of the Great Pyramid. Its perimeter, according to Mr Cole, is $4 \times 755{\cdot}8$ or 3023·2 ft. Earlier

* The same result follows of course approximately from the statement that the base of the Pyramid contains either 440 royal cubits and 500 Olympic cubits.

measurements made the base 760 ft. in length and the perimeter 3040 ft. Now the mean length of a nautical—meridian—mile, the 60th part of a degree, is 6080 ft. Thus the perimeter of the Pyramid is very approximately half a meridian mile. Did the wise men of old know this, and were they aware that the earth is spherical and about 8000 miles in diameter?

If the perimeter of the Pyramid was 2000 Olympic cubits then the Olympic cubit is 1/4000 of a meridian mile, and the Olympic fathom—the Greek *orgyia* of 4 cubits—is 1/60,000 of a degree.

It is all very curious; but can we credit such knowledge to the men who designed the Pyramids? They did place them very accurately with regard to the meridian: the faces of the Great Pyramid are only 3′ 43″ out, while the entrance gallery was directed* towards α Draconis, which was the Pole Star some 5000 years ago. Thales of Miletus, born B.C. 640, taught that the earth was spherical and occupied the centre of the Universe. Pythagoras, born B.C. 580, said that the sun was the centre of the planetary world and the earth one of the planets†. We know also that Muhammed Ibn Mesoud in his book (intituled in the Persian *Gchandanish*) relates that in the time of Ma' Mun‡, the learned Calif of Bagdad, by the elevation of the pole of the equator they measured the quantity of a degree upon the globe of the earth and found it to be 66 miles and two-thirds of a mile; every mile containing 4000 cubits and each cubit 24 digits and every digit 6 barley corns.

§ 5. THE ALEXANDRIAN TALENT

In connexion with the weights, equally interesting questions arise when one begins to ask how men in ancient days settled on their system of weighing. Since their standards of length represented without doubt the lengths of parts of their own bodies, their weights may well have been chosen to represent a definite quantity of some commodity of daily use, wheat or barley, the weight and length of whose grain has been in all countries intimately connected with standards of weight and length. The kilogramme was intended to be the weight of a unit of volume of water. How far back does this idea go? We have seen that the Alexandrian talent weighed 93·65 lb. or 655,550 grains. Now, taking the royal cubit as 20·62 in., the foot is 13·74 in. Thus the volume of a royal cubic foot is 2594 in.^{3}. Again, 1 lb. of water at 15° C. has a volume of 27·71 in.^{3}. Thus the weight of a royal cubic foot of water is 2594/27·71 lb. or 93·61 lb.

Is it a coincidence that the weight of the talent of Alexandria is so closely that of a royal cubic foot of water, or is the suggestion made by Mr Nicolson—that the agreement was designed—correct?

How much did the Wise Men of the East know?

* Another, possibly a more probable, explanation of the inclination of the gallery, about 20°, is that it is the angle of slope for brickwork.

† Greaves, *Discourse on the Roman Foot*, p. 193.

‡ Ma' Mun, the son of Haroun al Raschid, reigned from 808 A.D. to 833 A.D., and Muhammed Ibn Mesoud wrote about 1246 A.D.

The foot and the ounce. And here it is perhaps relevant to refer to another suggestion made by Mr Nicolson in an endeavour to account for the difference between the Olympic and the British foot. It is that the foot was chosen so as to make a cubic foot of water weigh 1000 oz. of (at the time) 437 grains each. For 1000 oz. contain 437,000 grains and a cubic inch weighs 252·62 grains. Thus the volume of 1000 oz. is 1729·9 in.^{3} and the cube root of this is 12,004 in., so that a cubic foot is very nearly the volume of 1000 Roman oz. of water at 15° C.

But I am digressing into curiosities of metrology. The weight of a cubic foot of water was investigated by the Oxford Philosophical Society in 1685. When Wren and the founders of the Royal Society moved to London, those of the members who were left behind at Oxford continued their meetings*, and at one of these two members, Messrs Caswell and Walker, described how they had made a hollow cube of very hard dry oak, each edge of which was very exactly 1 ft., with which they proposed to determine the weight of a cubic foot of water.

At a later meeting they communicated their result to the Society, finding it to be 1000 oz., and their result, along with the weights found for the same volume of a number of other substances, was sent to the Royal Society and is printed in the *Philosophical Transactions*† along with their table of specific gravities, which I have extracted.

Table 3. "A list of the Specific Gravitys of bodys, being in proportion as the following numbers."

Pump water	1000
Fir dry	546
Elm dry	600
Cedar dry	613
Wallnut tree dry	631
Crab tree meanly dry	765
Ash meanly dry, and of the outside lax part of the tree	734
Ash more dry, but about the heart	845
Maple dry	755
Yew of a Knot or root 16 years old	760
Beech meanly dry	854
Oak very dry, almost worm eaten	753
Oak of the outside sappy part, felld a year since	870
Oak dry, but of a very sound close texture	929
The same tryed another time	932
Logwood	913
Claret	993
Moil cider not clear	1017
Sea-water setled clear	1028
College plain Ale the same	1028
Urine	1030
Milk	1031
Box the same	1031
Redwood the same	1031
Sack	1033
Beer Vinegar	1034
Pitch	1150
Pit-Coal of Staffordsh,	1240
Speckled wood of Virginia	1313

* Their minute book is now at the Royal Society and from it these notes are taken.

† *Phil. Trans.* **169**, 926 (1685).

This has brought us very close to what I perhaps may call the more scientific period of our subject, and we may leave the speculations, interesting though they are, that centre round the Pyramids and their architects.

§ 6. THE ROYAL SOCIETY AND WEIGHTS AND MEASURES

We come now to more modern times, some two hundred years ago, and from that date onwards can trace with more certainty the history of our weights and measures.

In 1742 George II was on the throne; the Royal Society had been founded for nearly eighty years, and its Fellows were actively engaged in the pursuit of natural knowledge; the importance of exact measurement was becoming more fully recognized. Volume 42 of the *Transactions* for 1742–3 contains the account of various comparisons of weights and measures. There is also an account of the laying down of 3 ft. on a brass bar belonging to the Royal Society from a scale at the Tower.

Owing mainly to the action of Graham (November 11, 1742) sundry comparisons with French and other measures were made, and as a result he had a yard measure and a pound weight carefully made and examined. These were sent to Paris by the Society and, at the request of the French men of science, were compared with the French standards by M. du Fay and the Abbé Nollet. As the result it was reported that the half-toise of Paris contained 38·355 in. and the Paris foot 12·785 in., while the Paris two-marc weight of 16 oz. weighed 7560 Troy grains; thus the Paris ounce was 472·5 Troy grains, while an English avoirdupois pound weight belonging to the Royal Society contained 7004 grains, so that the English ounce was 437·52 grains. This seems to have led to the suggestion that a comparison of the existing standards of length and weight was desirable, and a committee consisting of Lord Macclesfield, Lord Clarendon, Mr George Graham and others, with Mr Cromwell Mortimer as secretary, undertook this in April, 1743.

Among the standards compared were a brass standard yard of Henry VII, marked with a crowned H., the Exchequer yard of Queen Elizabeth (now on the table—this is a square rod, an end standard), a yard from the Guildhall, London, another from the Tower, a yard belonging to the Clockworkers' Company and a sixth, then in the possession of the Royal Society.

The Committee reported (June 16, 1743) the differences shown in table 4, the Elizabethan yard being taken as standard.

Table 4. Comparisons of yards with the Exchequer standard of Elizabeth.

Royal Society yard	+ 0·0075 in
Henry VII yard	− 0·0071 in.
Guildhall yard	+ 0·0434 to 0·0396 in.
Clockworkers' yard	− 0·021 in.
Tower yard	+ 0·011 in.

Some standard ells also were measured and found to exceed 45 in. by from 0·04 to 0·05 in. The comparisons were made by the aid of a beam compass. The committee also examined a series of weights from the Exchequer and elsewhere.

As we have seen, from the time of Edward I the Troy pound is defined as containing 12 Troy oz. each of 20 pennyweights; no Troy standard pound was found among the Exchequer weights, but 12 oz. taken from a box containing 256 oz. weights were selected as representing the pound Troy. There was, however, a series of avoirdupois weights from 1 lb. to 14 lb. marked "Elizabeth Regina 1582," and a 7 lb. weight "VII El. 1588." Various comparisons led to the following results for the pound and ounce avoirdupois in Troy grains:

Pound	6998	7000·7	7000·2
Ounce	437·4	437·54	437·51

Of the weights in the possession of the Royal Society the avoirdupois pound was lighter than the standard by one grain, and the Troy pound lighter by half a grain.

This constitutes the earliest systematic comparison of our weights and measures that I have found. It was realized that the length of the standards varied with temperature, but it was held that since all were of much the same material the differences would not be affected.

Two standard bars were made by Bird in 1758 and 1760 respectively, for a committee appointed by the House of Commons under the chairmanship of Lord Carysfort. These were to be copies of the Royal Society standard as laid down by Graham 1742–3. One, marked "Standard Yard 1758" was presented to the House as the legal standard of length. Both were compared in 1768 with the R.S. standard, while in 1785 a scale made for General Roy's measurement of an arc of the meridian on Hounslow Heath was graduated in terms of the same standard.

Bills were presented to the House in 1765 to give effect to the recommendations of this and a subsequent committee, but from some accident they did not become law, and the country remained without legal standards for the next sixty years.

In 1792 Troughton* made a very full comparison of a standard bar constructed for Sir George Shuckburgh. In this comparison he made use of the R.S. Tower standard, the length marked on the R.S. bar from the Exchequer comparison by Graham (1743), two of Bird's bars, 1758 and 1760, Roy's bar, and the standards of Henry VII and Elizabeth. The modern method, employing micrometer microscopes, was used for the first time by Troughton in his comparison of the line standards; a beam compass was used for the end standards.

In 1814 a report was presented to the House of Commons by a new committee which asked Dr Hyde Wollaston and Prof. Playfair for advice. They stated that the length of the pendulum vibrating seconds had been found to be 39·13047 in. and that the metre of platina measured, at the temperature of 55°, 39·3828 English in., representing at 32° the ten-millionth part of the quadrant of the meridian. They remarked with great truth that although in theory the original standard of weight is best derived from the measure of capacity, yet in common practice it will generally be found more convenient to reverse the order, and they recommended, upon the

* *Phil. Trans.* **99**, 105–145 (1809).

suggestion of Dr Wollaston, "that a gallon containing ten pounds of pure water should be adopted as a substitute for the ale and corn gallons*."

In 1816 there still remained considerable doubt both as to the length of the seconds pendulum and the relation of the metre to the inch; the Astronomer Royal, Dr Pond, was instructed by Parliament to make the measurements necessary to remove the doubt. He asked for the assistance of the Royal Society and a committee† was appointed for the purpose.

A number of experiments were made by various methods‡, and in 1818 a commission§ was appointed under writ of the Privy Seal "to discuss the matter more minutely than could be done with convenience before a Committee of either House of Parliament."

The report‖ of this commission, drawn up by Dr Young, is of great interest; and as a result a bill giving effect to their conclusions was brought into the House of Commons by Sir George Clerk in 1822, and with a few amendments in 1823. This was nearly identical with an act which was passed on June 17, 1824, and came into force on January 1, 1826.

The commissioners at first recommended "for the legal determination of the standard yard that which was employed by General Roy in the measurement of a base at Hounslow Heath." This was afterwards altered to Bird's standard of 1760 in consequence of errors found by the committee in the Roy standard. The act enacted "that the straight line or distance between the centre of the two points in the gold studs in the straight brass rod now in the custody of the Clerk to the House of Commons whereon the words and figures Standard Yard 1760 are engraved shall be and the same is hereby declared to be the original and genuine standard of that measure of length or linear dimension called a yard." This distance, when the rod was at a temperature of 62° F., was denominated the "Imperial standard yard" and all other measures of extension were to be multiples or sub-multiples of this standard.

Thus the standard finally chosen was Bird's standard of 1760: it was found to agree closely with Sir George Shuckburgh's scale, with which the length of the pendulum and the metre had been compared, and of which a facsimile was known to exist in Geneva¶.

For the standard of weight the commissioners recommended "that the standard brass weight of two pounds Troy weight now in the custody of the Clerk of the House of Commons shall be considered authentic," and it was proposed "that a brass weight equal to one half of the said brass weight of two pounds gravitating in

* This quotation is from the article "Weights and Measures," by Dr Thomas Young in the seventh edition of the *Encyclopædia Britannica* (1842), to which I am indebted for much information.

† The committee consisted of the president and secretaries, Sir Chas. Blagden, Mr Gilbert, Dr Wollaston, Dr Young, Capt. Kater, General Mudge, Mr Brown, Mr Rennie and Mr Troughton.

‡ Capt. Kater's pendulum work is described in the *Phil. Trans.* for 1826, 1830 and 1831.

§ The commissioners were: Sir Joseph Banks, Sir George Clerk, Mr Davies Gilbert, Dr Hyde Wollaston, Dr Thomas Young and Capt. Henry Kater. Dr Young "with the assistance of a clerk who had studied the law" undertook the duties of secretary.

‖ The more important parts are printed in Appendix 1, p. 454.

¶ Sir Geo. Airy, *Phil. Trans.* (1857).

air (the barometer being at thirty inches and the thermometer being at 62° by Fahrenheit's scale (1822))* shall be, and the same is hereby declared to be the original and genuine standard of weight; and that such brass weight shall be and is hereby denominated the Imperial Standard Troy Pound." This Troy pound was declared to contain 5760 grains "and 7000 such grains shall be and are hereby declared to be a pound avoirdupois."

For the recovery of the standards if lost, destroyed, defaced or otherwise injured, suitable provision was made in the act; the yard was to be recovered from the length of the seconds pendulum which, when vibrating in a vacuum at sea level in the latitude of London, was stated to be 39·1393 in., while it was held that the pound could be reconstructed from the fact that a cubic inch of distilled water weighed by brass weights in a vacuum at a temperature of 62° had been found to weigh 252·724 grains. It will be noticed that, for the purpose of recovery, recourse is to be had to the length of the pendulum and the weight of a cubic inch of water, not to the old measures from which the new Imperial standards had been derived or to any copies of those standards which might be in existence.

§ 7. THE MODERN STANDARDS

The need for reconstruction came all too soon. In 1834 the Houses of Parliament were destroyed by fire; the pound standard was lost entirely and the yard standard was damaged, one of the gold plugs having been melted so that its centre point could no longer be determined with accuracy. In 1838 the Chancellor of the Exchequer, the Rt. Hon. T. Spring Rice, set up a commission† to advise as to the steps necessary to replace the standards.

Meanwhile, since 1824 a number of criticisms of the means suggested for recovering the standards had been made. Certain of the corrections employed by Kater in determining the length of the seconds pendulum were shown to be in error; while the agreement among the various determinations of the weight of a cubic inch of water left the true value in doubt, and the commission‡ of 1838, accepting these criticisms, threw over the methods provided in the act of 1824 and advised, with regard to the pound, that it should be recovered by comparison with copies which were known to exist and that the pound avoirdupois should be the standard. As to the yard, it was stated that "Several measures now exist which were most accurately compared with the standard yard... and by the use of these the values of the original

* In 1823 this became "That the Standard brass weight of one pound Troy weight made in the year 1758 now in the custody of the Clerk of the House of Commons."

† Full accounts of the work of this commission and of the scientific commission which followed it in 1843 will be found in the following two papers in the *Philosophical Transactions of the Royal Society*: (1) Prof. W. H. Miller, "On the construction of the new Imperial Standard pound and a comparison with the Kilogramme des Archives," *Phil. Trans.* **146**, 753–946 (1856). (2) G. B. Airy, Esq., Astronomer Royal, "Account of the construction of the new national standard of length and of its principal copies," *Phil. Trans.* **147**, 621–702 (1857).

‡ This commission consisted of the Astronomer Royal (Mr G. B. Airy), Mr F. Baily, Mr Bethune, Mr Davies Gilbert, Sir J. F. W. Herschel, Mr Lefebvre, Mr J. W. Lubbock, Rev. Geo. Peacock, and Rev. R. Sheepshanks.

standards can be respectively restored without sensible error." The commission therefore recommended a material standard, "the distance between two points or lines engraved on metal," "but that the standard be in no way defined by reference to any natural basis such as the length of a degree of a meridian on the earth's surface in an assigned latitude or the length of a pendulum vibrating seconds in a given place."

The Government approved this recommendation and appointed a scientific committee in 1843 to reconstruct new parliamentary standards of length and weight.

The pound. Prof. Miller was entrusted with the construction of the pound, Mr Baily with that of the yard. Prof. Miller has described his work very fully in the Royal Society paper already referred to. He gives a list of the Troy pounds, fourteen in number, some of platinum, others of brass or gun metal, available for comparison, and describes in detail the balance employed and the method of weighing (Gauss's method by double weighing was used). A thermometer, K. 43, was specially calibrated at Kew Observatory for the work, and great precautions were taken to secure accuracy. The standard weight of 1824 was known as U. It had been compared in 1824-5 by Capt. Kater with five gun-metal standards deposited at the Exchequer, the Royal Mint, and with the civic authorities in London, Edinburgh and Dublin.

In 1829, U had been compared by Capt. V. Nehus with a platinum pound (R.S.) belonging to the Royal Society, and also with two brass pounds and a platinum pound (Sp.) in the custody of Prof. Schumacher*. These weights were available for the construction of the new pound. The comparison showed considerable discrepancies in the values of the gun-metal and brass weights and in the end it was resolved "with the consent of the Astronomer Royal to rest for the evidence of the weight of the lost standard entirely on the comparisons of the two platinum standards Sp. and R.S."

The density of the lost standard was not known with accuracy, and this led to uncertainty. A value deduced from observations on weights of the same date as the standard (1758) was employed in the reduction of the weight to that in a vacuum.

In the course of the work quartz was examined as a material for the standard but the conclusion was reached that it was insufficiently dense, and the corrections required to the weight in air were in consequence unduly uncertain.

As the result of this work and the corresponding work on the yard our present standards came into existence, and the Weights and Measures Act of 1855 prescribes that: "Whereas by Act of the Fifth Year of King George IV a standard brass weight of one pound Troy made in the year 1758 then in the custody of the Clerk of the House of Commons should be...the genuine standard of weight...and whereas there exist weights which had been accurately compared with the said standard pound Troy which afforded sufficient means for restoring such original standard, the said weight of platinum marked 'P.S. 1844 1 lb.,' deposited in the Office of the

* Schumacher was Professor at Copenhagen. He became a Foreign Member of the Royal Society in 1821 and in 1836 communicated to the Society a paper, "A comparison of the late Imperial standard Troy pound weight with a platina copy of the same and with other standards," *Phil. Trans.* **126**, 487–495 (1836). His portrait hangs in our Library.

Exchequer as aforesaid, shall be and be denominated the Imperial standard pound avoirdupois. The one 7000th part of this shall be a grain and 5760 such grains shall be a pound Troy."

Provision is made in the act for recovery of the standard, if lost, by comparison with copies made and distributed among various authorities.

Thus the pound avoirdupois became our standard of weight and the pound Troy followed in the path of the King's Tower pound.

With regard to the comparison between the pound and the kilogramme, Prof. Miller writes, "The comparison left me in entire darkness as to the real value of the kilogramme." The standard kilo of the French Bureau denoted by $\sigma\tau$ was, by the kindness of Arago, compared with the pound with the result that

$$\sigma\tau = 15432{\cdot}3489 \text{ grains}^* = 2{\cdot}2046 \text{ lb.}$$

A platinum kilo ϵ obtained for England was compared with $\sigma\tau$ with the result that

$$\epsilon = \sigma\tau - 0{\cdot}02435 \text{ grains.}$$

But at the time the density of $\sigma\tau$ had not been determined and to this extent the result was uncertain.

The yard. Mr Baily, at the request of the committee, undertook the reconstruction of the yard. The account of the work is given by Sir George Airy in the paper of 1857 already referred to.

In addition to the damaged standard, the following were available for comparison: a scale, No. 46, belonging to the Royal Society, a scale belonging to the Royal Astronomical Society, two three-foot bars of the Ordnance Society. There was also the Shuckburgh scale of 1792 but, on account of certain imperfections, little importance was attached to it.

The standard was to be of bell metal or steel, a rectangular bar about 1 in. square in section, with the ends notched away to half the thickness. The marks defining the length were to be on plugs inserted flush with the surface and the standard was to be correct at 61° or 62° F. The ends were cut away in order that the marks might be in the median plane of the bar, and the effect of flexure when supported at two points minimized.

Baily began by making and testing the suitability for his purpose of various alloys; he finally settled on one, since known as Baily's metal, consisting of copper 16 parts, tin $2\frac{1}{2}$ and zinc 1 part. Of this the standard was ultimately constructed.

Faraday was consulted somewhat later as to the stability of such an alloy, and wrote, "I do not see why a pure metal should be particularly free from internal changes...I suppose the labour would be too great to lay down the standard on different metals and substances, and yet the comparison of these might be very important hereafter, for twenty years do seem to do or tell a great deal in relation to standard measures."

Baily also introduced the method of drilling a hole in the bar down to the median

* The equivalent legalized by the Weights and Measures Act of 1897 is 15432·2564 grains or 2·2046223 lb.

section, and fixing the plug with the mark at the bottom of this instead of notching the end as Kater had done. His work was cut short by his death in August, 1844.

At this stage Sheepshanks offered to take Baily's place, and the question immediately before the committee was whether to attempt to restore the damaged legal standard or to construct a new standard from the available material based on Shuckburgh's yard which Kater had considered satisfactory. It was plain that in the strictly legal or scientific sense the restoration of the standard must be indefinite, and the latter plan was approved. Sheepshanks suggested that the necessary apparatus for his comparisons should be set up in the cellar of Somerset House and made a report on the questions involved—the stability of the material, the thermometric difficulties, etc., as well as on the method he proposed to adopt.

A meeting of the committee was held on June 4, 1847, to inspect these arrangements, and the minutes record that "The Committee approve entirely of the course followed by Mr Sheepshanks and request Mr Airy to take measures for the exam^{n} and discharge of the instrument makers accounts."

Airy's paper already referred to gives very full details of the construction of the standards; it also contains his investigation of the effect of flexure and the determination of best points of support.

Ultimately six standards of Baily's metal were constructed which were found to have the length of 1 yd. at the temperatures indicated below:

No. 1.	Correct at	62°	F.
2.	,,	61·94°	F.
3.	,,	62·10°	F.
4.	,,	61·98°	F.
5.	,,	62·16°	F.
6.	,,	62°	F.

Of these it was arranged that No. 1 should become the standard yard, while 2, 3, 4 and 5 should be known as parliamentary copies and No. 6 should be retained by some officer of the Government for the comparison of other standard bars or for other scientific purposes.

Sheepshanks was taken ill while actually engaged in his comparisons; he died the day before royal assent was given in 1855 to the act establishing the new measures. Thus, as a result of the death of the two men who had done most of the work, the Royal Society paper in which an account of it is to be found was written by Airy. The act of July 30, 1855, provides that "The straight Line or Distance between the Centres of the two Gold Plugs in the Bronze Bar deposited in the Office of the Exchequer as aforesaid shall be the genuine standard of that Measure of Length called a Yard and the said straight Line or Distance between the Centres of the said Gold Plugs or Pins in the said Bronze Bar (the Bronze being at a temperature of Sixty two Degrees by Fahrenheit's Thermometer) shall and be deemed to be the Imperial Standard Yard." Thirty-seven copies were made and distributed. Such were the conclusions enacted in the Weights and Measures Act of 1855, and they have remained unaltered up to the present time.

Since that date there have been various supplementary or consolidating acts. In 1864 The Metric Weights and Measures Act was passed. It provided that "No contract...shall be deemed to be invalid or open to objection...on the ground that the Weights and Measures expressed...are Weights or Measures of the Metric System," and it gives a schedule of tables of metric measures expressed by means of legalized denominations of weights and measures in Great Britain, thus legalizing the metric system but not providing any metric standards for purposes of comparison and verification.

In 1866 the custody of the imperial standards of length and weight and of all secondary standards was transferred by act of Parliament to the Board of Trade. Parliamentary copies of the yard and pound were placed at the Royal Mint, with the Royal Society and at the Royal Observatory, Greenwich, and it became the duty of the Board to cause these to be compared every ten years with the standards in their charge. For this and other purposes connected with standards the Board were to constitute a department to be called the Standards Office and "shall appoint as Head of that Department an Officer to be called the Warden of the Standards" with Assistants, Clerks, etc. The act continues, "It shall be the duty of the Warden of the Standards to conduct all such comparisons...or other operations with reference to the Standards of Length Weight or Capacity in aid of Scientific Researches or otherwise as the Board of Trade from time to time authorize or direct." It was also the duty of the Warden to make an annual report to the Board to be laid before both Houses of Parliament.

The procedure was modified by an act of 1878 relating to the standards deposited in the Standards Department of the Board of Trade in the custody of the Warden of the Standards which provides (para. 33) that "The Board of Trade shall have all such powers and perform all such duties relating to Standards of Weight and Measure as are by any Act or otherwise vested in or imposed upon" various official persons "or the Warden of the Standards," and it was made the duty of the Board of Trade to conduct all such comparisons as the Board of Trade from time to time thinks convenient and to report to Parliament.

Other sections of the act definitely transfer to the Board all the various duties assigned by the act of 1866 to the Warden of the Standards.

The act also gives a schedule of metric equivalents and a list of metric standards in the custody of the Board and provides (para. 38) that "Whereas the Board of Trade have obtained accurate copies of the Metric Standards and it is expedient to make provision for the verification of Metric Weights the Board of Trade may, if they think fit, cause to be compared with the Metric Standards in their custody all metric weights and measures submitted for the purpose."

A Metric Convention (Convention du Mètre) had been agreed to among a large number of nations in 1875. To this Great Britain adhered in 1884, and comparisons between the metric and British measures were set on foot. The work was carried out by Mr Chaney of the Standards Department of the Board of Trade and M. Benoît, Director of the Bureau International des Poids et Mesures, and in 1889 certified copies of the metre and kilogramme were deposited at the Board of Trade.

In 1894 the International Committee of Weights and Measures suggested the desirability of a comparison between the standard yard and the metre. This was agreed to. Mr Chaney made a careful comparison between the standard and the parliamentary copy No. 6 which was then sent to Sèvres and compared with the metre, by M. Benoît.

A yard is about 914 mm. in length and the outstanding difficulty in any such comparison is the subdivision of the metre to make sure of the length marked as 914 mm. Special means were taken to determine the difference between the two standards accurately in terms of the inch, with the result that the value 1 metre = 39·370113 inches was finally accepted by both investigators and became the legal equivalent under the act of 1897.

As the result of these comparisons a further act was passed in 1897 giving to the Queen in Council power to make a table of metric equivalents in substitution for the table in the schedule to the act of 1878, and providing that the Board of Trade standards which may be made under para. (8) of the act of 1878 shall include standards from the metre No. 16 and the kilogramme No. 18 deposited with the Board of Trade.

Under the provisions of this act a table* of metric equivalents was issued in 1898 according to which

$$1 \text{ metre} = 39{\cdot}370113 \text{ in.}$$

$$1 \text{ kilogramme} = 2{\cdot}2046223 \text{ lb.}$$

The most recent comparison† of the yard and metre is that made at the National Physical Laboratory in 1927 which leads to the result that

$$1 \text{ metre} = 39{\cdot}370147 \text{ in.}$$

$$1 \text{ inch} = 25{\cdot}399956 \text{ mm.}$$

Here perhaps a reference to the Metric Convention‡ is desirable. An International Metric Commission was convened at Paris in 1872 to consider the question of the construction and issue of metric standards of weight and measure. A committee was appointed to make the necessary preliminary investigations, and as the result a convention was agreed to on May 20, 1875, by a large number of States. Great Britain adhered to the convention in 1884. The convention provided that the contracting parties should set up at Sèvres an International Bureau of Weights and Measures at which the international prototypes of the kilogramme and metre were to be kept. For the control of the bureau an International Committee of Weights and Measures, consisting of fourteen members belonging to different countries, was established. The committee was placed under the authority of a General

* *Statutory Rules and Orders*, 1898, No. 411.

† J. E. Sears, Jr., C.B.E., M.A., W. H. Johnson, B.Sc., H. L. P. Jolly, M.A., "A new determination of the ratio of the imperial standard yard to the international prototype metre," *Phil. Trans.* A, 227, 281–315 (1928).

‡ International Convention of 1921 modifying the International Convention of May 20, 1875, Stationery Office, Treaty Series No. 24 (1923), Cmd. 1982.

Conference of Weights and Measures formed of delegates from all countries adhering to the convention. The conference is to meet whenever convoked by the international committee, and in any case once in every six years*. The president of the French Academy is the president of the international committee. The first duty of the committee was to make and distribute to the adhering countries copies of the prototype standards; while this work was continuing† meetings were to be held annually. At the conclusion of the work the meetings became biennial.

The prototype standards at the bureau are under the charge of the committee‡ and a special resolution of the committee is required to permit of their examination. The first Director of the Bureau was M. Benoît. The present holder is M. C. E. Guillaume, who has done so much for accurate metrology and who delivered the Guthrie Lecture in 1920.

Inter-dominion standards. Before we leave this part of our subject—the present position of our national standards—a brief reference should be made to the Conference§ on Standardization held at the Board of Trade in October, 1930, under the Chairmanship of Mr W. R. Smith, M.P., Parliamentary Secretary to the Board. The conference was attended by representatives of the United Kingdom, the Dominions of Canada, Australia and New Zealand, the Union of South Africa, the Irish Free State and India. A representation from the Office of the Crown Agents attended as an observer on behalf of the non-self-governing parts of the Empire. The work of the conference was divided into two parts relating respectively to fundamental standards and to industrial standardization. Sir Joseph Petavel acted as chairman of the section dealing with fundamental standards.

The Imperial Conference accepted the report of the Conference on Standardization and adopted the resolutions submitted therewith.

These resolutions, so far as they deal with fundamental standards, were:

(1) "That it is desirable that there should be uniformity between the standards employed for all units of measurement which are in common use among the British Commonwealth of Nations. (2) In order to secure uniformity arrangements should be made (*a*) to provide in each Dominion and in India suitable reference standards for each unit of measurement required for use in that country where not already available; and (*b*) to introduce suitable procedure whereby all such standards shall be periodically compared with the corresponding standards at the Board of Trade or at the National Physical Laboratory. (3) At least one member of the Commonwealth should undertake research work with the object of enabling the fundamental standards to be referred ultimately to natural standards such as the wave-length of light. It would be a great advantage if it were possible for research work of this character to be carried out independently by more than one member."

* The last meeting was in 1930. Mr Sears was the British delegate.

† This task was completed in 1889.

‡ A brief notice of the recent activity of the international committee in connexion with electrical standards will be found on p. 447.

§ Report of the Conference on Standardization (including resolutions adopted by the Imperial Conference). Presented to Parliament by command of His Majesty, November, 1930. Cmd. 3716–1930.

The report of the Fundamental Standards Committee defines the fundamental standards, both on the British system and on the metric system, which constitute the fundamental units of measurement legal for use throughout the British Commonwealth of Nations. It is noted that in India the railway system of weights in common use is defined in terms of the pound. It is explained that the term "reference standard" is used to denote a standard of suitable form and construction preserved under suitable conditions to serve as the basis of reference for the determination of any unit of measurement within one of the Dominions or of India. In accordance with the recommendation of the report the errors, if any, of such reference standards in relation to the fundamental standards or to the theoretical definitions of the units they represent are to be ascertained from time to time by comparison with the corresponding standards in use in the United Kingdom, and are to be allowed for in the use of the reference standards. The report concludes with a list of the principal units for which reference standards are likely to be required, with suggested procedure in regard to each.

Thus a simple and definite system has been set up by the Imperial Conference of 1930 for securing uniformity of standards throughout the Home Country, the Dominions and India.

§ 8. TIME

I have dealt at some length with the standards of length and mass. Time is for the physicist the third fundamental unit, and a lecture on standards of measurement would be incomplete without some reference to it. But the reference must be very brief; the subject is astronomical, and to attempt any full account of the measurement of time from the shadow clock or water clock of the Egyptians to the free pendulum of Mr Shortt or the vibrating crystal developed at the Bell Laboratories* in New York by Mr Marrison is a task far beyond my powers.

The apparent diurnal motion of the stars about the pole of the equator, the monthly motion of the moon, the yearly motion of the sun among the stars are the phenomena on which the measurement of time has always been based. Early observation showed that the moon returned to the same position among the stars in about thirty days—a month—while in twelve of these months—360 days—the annual cycle of the sun was, it appeared, complete. In each day the sun moved through 1/360th of its path, and this may have led to the division of the circle into 360 degrees. Division by six, or six multiplied by some power of ten, was very common in ancient measurements, while the sixths were sometimes halved, leading to division by twelves, e.g. the ounce and the inch. Thus the day—a day and a night—was divided into two equal periods, each containing twelve hours, while the hour was divided into sixty minutes and each minute into sixty seconds.

In quite early times, however, it was realized that the year contained about 365 of these days of twenty-four hours and "by attentively observing the heliacal rising of the star Sirius to which they gave the name of Thaat or Thoth (the Watch

* W. A. Marrison, "High Precision Standard of Frequency," Bell Telephone Laboratories, August, 1929, B 407.

Dog), because its appearance shortly preceded the overflow of the Nile, the Egyptians had discovered that the year consists of $365\frac{1}{4}$ days*." This was known to the Chinese, who in the time of the Emperor Yao, about 2300 B.C., divided the circle also into $365\frac{1}{4}$ degrees. The Chaldaeans in the far past discovered the cycle of 223 lunations, eighteen solar years, which bring the sun, earth and moon into approximately the same relative positions and so give rise to a repetition of the series of eclipses.

Shadow clocks were used at a very early date. The earliest known reference to a sundial† is to that of Berossus, a Chaldaean astronomer of about 300 B.C. One of these was erected in Rome about 290 B.C., and another found at Pompeii was constructed for the latitude of Memphis.

Water clocks were used in Egypt, the time being measured by the change of level in the water contained in a suitable reservoir from which it escaped by a small aperture. It is said‡ that wheel-work was employed and the hours shown on a graduated scale.

Ibn Janis and other Arabian astronomers about the year 1250 A.D. were in the habit of measuring small intervals of time by the oscillations of a pendulum which they kept in motion by a light touch of the finger when the vibrations had become too small to be easily counted. They did not connect the pendulum with a train of wheels.

A bar weighted at each end and kept in oscillation about its centre by the action on two pallets of the teeth of a wheel, maintained in continuous rotation by a spring or weight, was used to secure uniformity of motion of the wheel and thus to mark time. An early clock made about 1364 by Henry de Wyck for Charles V of France was regulated thus. *The Times* of May 9 last contains an account of a clock in Salisbury Cathedral dated about 1386 which has recently been cleaned and re-erected in the north transept.

Later the balance bar became a wheel with a heavy rim, while later still, in 1638, Hooke added a spiral spring to regulate the motion of the wheel. This, as it coiled and uncoiled, communicated to the wheel an oscillatory motion of a definite period. The motion of the balance wheel was maintained by a weight or a spring acting through a train of wheels, the last wheel of the train being connected to the balance by the escapement, a device which allowed one tooth of the wheel to pass at each oscillation of the balance and thus secured uniformity of motion in the train of wheels.

In 1600 Galileo called attention to the isochronous character of the motion of the pendulum. In 1633 he suggested its application to a clock; the problem was more fully investigated by Huyghens in 1638. Huyghens studied the relation between the period and the arc of vibration, and proved that for isochronism the

* *Encyclopædia Britannica* (*Seventh Edition*): Art. "Astronomy."

† The word translated "sundial" in the reference (Isaiah xxxviii. 8) to the "sundial" of Ahaz, B.C. 700, should be "steps"; the translation from the Hebrew is incorrect. *Encycl. Biblica.*, Art. "Dial."

‡ Dr Thomas Young, *Lectures on Natural Philosophy*, Lecture 17.

arc must remain constant unless the motion was compelled to be cycloidal. To maintain the arc constant a small impulse must be given to the pendulum at stated intervals—theoretically at each swing—and it was early recognized that this should occur as the pendulum is passing through its lowest—zero—position. Endeavours to secure this condition led to modifications of the escapement, the most important of which were due respectively to Harrison and to George Graham, who about 1700 introduced his dead-beat escapement which was almost universally used on astronomical clocks until, about 1890, Riefler of Munich produced a still more accurate mechanism.

Graham and Harrison also each introduced means to compensate for the change of length of the pendulum due to changes of temperature. One or other of these, usually Graham, was used until Guillaume's discovery of invar.

Electric clocks. The idea that the necessary energy could be communicated to the pendulum by electromagnetic forces seems first to have occurred to Alexander Bain, a mechanic in the employ of Sir Charles Wheatstone, who in 1840 took out a patent for the purpose. Since that date some eight hundred such patents have been issued. Prof. Sitter of Leyden, writing in *Nature* in 1928* of the Shortt clock, patented by Mr W. H. Shortt in association with the Synchronome Company, stated: "One of these clocks has been left entirely to itself, being however kept under rigorous observation at Greenwich during the greater part of a year, and its rate has been absolutely invariable....It looks as if this clock could be depended to keep time within a few hundredths of a second for a period measured in years instead of days." The pendulum of a clock has ordinarily two duties to perform. (1) It must execute its oscillations in equal periods, receiving in some way, from a suitable source, just enough energy to maintain the constancy of the arc of oscillation, without in any way else affecting its motion. (2) It must secure uniformity of motion in the train of wheels which constitute the works of the clock and indicate time by the motion of the hands.

Now the good time-keeping of a pendulum depends mainly on its being left alone to swing freely under gravity, receiving just enough energy to maintain its motion, but not being called upon to control directly the motion of a complex mechanism. In the Shortt clock this is secured by entire separation of the two functions of the pendulum described under (1) and (2) above. There are two pendulums; the one, the master, is free, the second known as the slave, which by suitable means is made to have exactly the same period as the free pendulum, controls the mechanism of the clock. The free pendulum swings in an air-tight case kept at greatly reduced pressure and at constant temperature. The swings of the slave are identical both in period and in phase with that of the master pendulum. Each half-minute—not each beat—as the two pendulums, moving from right to left (figure 3), approach the zero of their swing, a contact made by the slave allows a current to pass round an electromagnet in the case of the master clock, thus causing an arm, carrying a jewel at its free end, to fall on a wheel attached to the free pendulum, which is just then at the centre of its swing. The weight of this arm pressing on the

* *Nature* **121**, 99–106 (1928).

right-hand side of the wheel gives the pendulum the energy it has lost during the previous half-minute. As the arm falls off the wheel it makes a contact, actuating one electromagnet, which replaces the fallen arm in position ready for the next half-minute impulse and, at the same time, a second electromagnet controlling a spring which can act on the slave pendulum. If the slave clock is more than 1/250th of a second slow this spring comes into action and quickens the clock by this amount. The slave pendulum controls the clockwork and does all the work. Figure 3 is a diagram, taken from Mr Hope Jones's* book, which indicates the connexions. Such is a very brief description of the most remarkable clock of ancient or modern times.

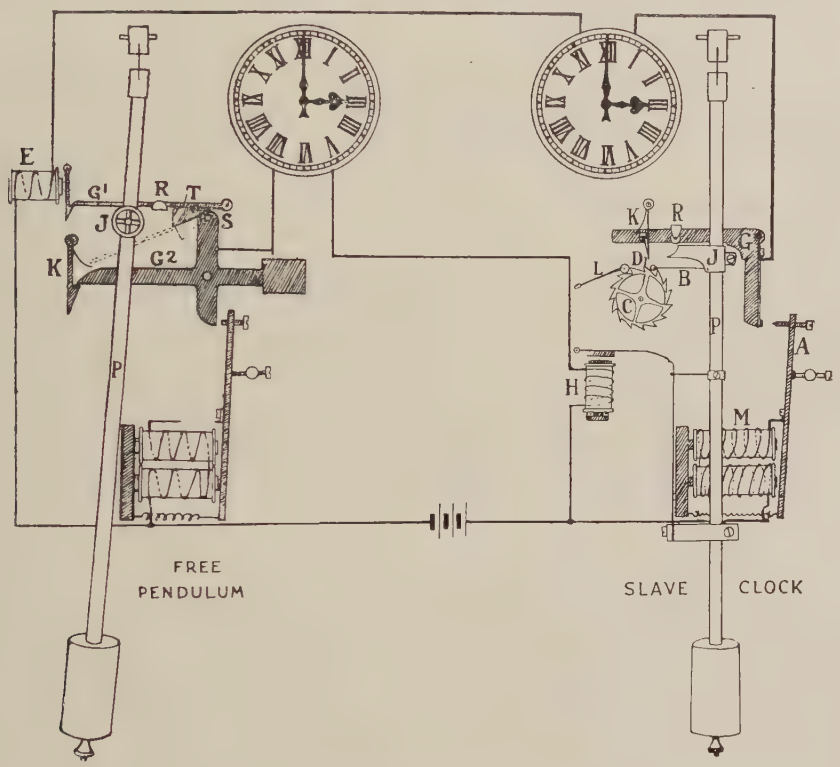


Fig. 3. The Shortt clock.

By permission of Mr Hope Jones.

The vibration clock. But the various methods of measuring time, which have been referred to, all depend on the uniform rotation of the earth for the calibration of their record over one or more days, and the uniformity of gravity or of the elastic forces brought into play by the deformation of a spring for its divisions into smaller units, hours, minutes and seconds. Now a body such as a bar of steel or a block of quartz, when set into vibration, has a number of frequencies depending on its shape, the nature of the motion, and the molecular properties of the material. The fre-

* I am indebted to the Astronomer Royal and to Mr Hope Jones for descriptions of the clock and for permission to include the figure.

quency of these is invariable so long as the body remains in the same physical state. It is possible under certain conditions by the action of an oscillating electric force to set such a body into vibration and to arrange that the body when in vibration reacts on the electric force producing the motion, in such a way as to reinforce its own vibration and thus maintain itself in an unvarying condition of motion of constant period. Such a system is not affected by the earth's rotation; the frequency remains constant so long as the physical conditions affecting the system are constant, and if the number of vibrations in an interval of time, measured by a clock, can be counted we have a means of checking the accuracy of the clock's record.

Recently a number of quartz oscillators have been set up at the Bell Telephone Laboratories in New York by Mr W. A. Marrison and the following brief account of these is taken from his paper*. "Plates of quartz cut in one direction from a crystal have a positive temperature coefficient, while other plates cut in a different direction have a negative coefficient. It is thus possible to cut a plate or ring from the crystal in which the frequency is independent of small variations of temperature.

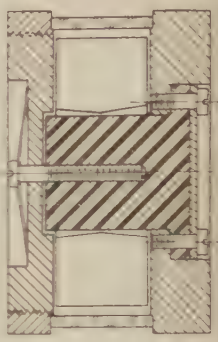


Fig. 4. Section of crystal mounting showing point support.

"A ring of the crystal, figure 4, is supported on a horizontal cylinder between two electrodes which are kept at a fixed distance apart by a ring of pyrex glass, and the hole in the crystal is so shaped that the line of contact with the cylinder is at a node for the two vibrations set up by the electric forces. There are arrangements for keeping the crystal in a fixed position relative to the electrodes and the whole forms part of a tuned oscillating electric circuit, the electrodes being connected to the earthed pole of the battery and the grid of the lamp respectively. The apparatus is very carefully lagged to reduce variations due to temperature to a minimum."

An account of a long series of comparisons between the records of these oscillators and three Shortt clocks in the laboratory of Mr A. L. Loomis at Tuxedo, U.S.A., was given to the Royal Astronomical Society by Mr Loomis on March 13, 1931†. Mr Loomis's paper contains a description of the very beautiful chronograph used to make the records. A comparison of the records made at Tuxedo was carried out by Prof. E. W. Brown and Mr Brouwer and the results were given at the same meeting of the Royal Astronomical Society. The examination showed that in any period of 30 seconds variations of as much as 0·001 second were rarely present. The records also show a very small variation in the period of the clock which was traced to a change in the value of g due to the influence of the moon‡, agreeing almost exactly with the amount expected by calculation.

During the past few years Dr Dye has carried out at the National Physical

* *Bell Telephone Laboratories Journal*, **8**, 493 (1929).

† *Monthly Notices R.A.S.* **91**, 569–575 (1931).

‡ This brings back to my remembrance and possibly to that of one or two of my hearers the attempt made in the very early days of the Cavendish Laboratory, now more than 50 years since, by George and Horace Darwin to investigate this effect, an attempt which ended in failure.

Laboratory a large amount of work on the quartz oscillator as a standard of frequency, investigating the conditions for maintenance of the oscillations as well as the influence on the frequency and power output of variations in the circuits associated with the quartz. Reference to this work will be found in the recent Reports of the Laboratory.

The same problem has been attacked in the Metrology Department of the Laboratory. Mr Sears utilizes for his vibration clock the longitudinal vibrations of a bar a metre in length held at its nodal point. This provides a very pure mode of vibration, and with a close control of the temperature it is hoped to realize a very high degree of isochronism. He has kindly furnished me with the following brief account of the arrangement as worked out by Mr Tomlinson:

"The bar is of elinvar and is self-maintained by means of electrostatic forces. Each end is highly finished, flat and square to the axis, and is associated with a small parallel fixed plate at a distance of about 0·003". These form small condensers, in one of which a feeble alternating potential is induced by the motion of the bar. This is amplified and applied as a driving potential to the other end. The electrostatic method of maintaining the vibration is expected to reduce the reaction on the frequency due to variations of the circuit to a minimum much less than would be attainable with a magnetic method of driving.

"To count the total number of vibrations, a phonic motor of special design runs in synchronism with the bar, driving a registering clock train. The first spindle of this train is arranged to rotate approximately once per second and is fitted with a contact device by means of which seconds signals, very precisely defined, are recorded by one marker of a special chronograph. A second marker records the duration of any time interval to be measured by comparison with the vibration clock.

"The temperature-control of the vibrating bar will be obtained by enclosing the bar completely in a steel vessel which is immersed in a large water bath, with a vigorous circulation kept up by a propeller. A toluene thermostat submerged in the bath regulates the temperature by controlling the current supplied to a group of electric heaters. The latter details are all that remain to be completed before the clock is ready for trial."

A chronograph for use with the vibration clock has been made and tested. An account of this appears in the *Journal of Scientific Instruments**. In a trial of the chronograph the small irregularities shown in a series of 10-second intervals obtained from the vibration clock did not exceed 0·0002 sec. When the chronograph was being used a period of 3 min. could be measured with an accuracy of about one part in a million. This very brief and incomplete account of the history of time-measurements is perhaps sufficient to illustrate human progress from the stage when the year was thought to contain 360 days to the present time, when a thousandth of a second can be measured with accuracy and the pull the moon exerts on a penny can be recorded on a clock.

* J. E. Sears, Jr., C.B.E., M.A., and O. A. Tomlinson, B.Sc., "A high precision chronograph," *J. Sci. Inst.* **8**, 21–28 (1931).

§ 9. ELECTRICAL STANDARDS

To turn now to electrical standards. There is neither time nor space for a long discussion. Besides, eighteen years ago, I delivered the fourth Kelvin Lecture* before the Institution of Electrical Engineers; it occupies, I find, thirty-three pages of their *Journal*. In that I tried to say all that I knew about electrical standards and to it I must refer you for many details. All I need do now is to give a brief summary of those first fifty years with some reference to the work of the eighteen which have followed.

Coulomb in 1785 had shown how, by the use of the proof plane and the torsion balance, a quantity of electricity could be measured, and had verified the inverse-square law from which we now derive the electrostatic unit of electricity. Faraday in 1834 discussed the "definite chemical action of electricity†." He had shown in December, 1832, "when experimenting with a battery of Leyden jars and a galvanometer that the deflecting force of an electric current is directly proportional to the absolute quantity of electricity passed," and further that "the chemical power, like the magnetic force, is in direct proportion to the absolute quantity of electricity which passes," two experimental results on which depend the electromagnetic and the electrochemical measures of a current.

But it was not until 1851 that W. Weber made his first distinct proposal of a definite system of electrical measurements according to which resistance would be measured in terms of an absolute velocity‡."

Various suggestions for a standard of resistance had been made previously to 1861 when the British Association, at the suggestion of Sir William Thomson, appointed a committee for improving the construction of practical standards of electrical measurements. The appointment of the committee was the outcome of a paper by Sir Charles Bright and Latimer Clark proposing names for the standards of resistance, current, electromotive force and quantity.

In the *Electrician*, in which this paper is printed, the editor writes: "We fear there is some danger that a system may be devised which will be followed exclusively by the eminent gentlemen at whose suggestion it is put forward." While Latimer Clark, in a letter published shortly afterwards, expressed the "fear that while bringing the highest electrical knowledge to the subject and acting with the best motives they (the committee) may be induced simply to recommend the adoption of Weber's absolute units or some other units ill adapted to the peculiar and various requirements of the electric telegraph." His paper had proposed the e.m.f. of a Daniell cell as a unit of electromotive force, and the charge on an air condenser of two parallel plates 1 m.^{2} in area and 1 mm. apart, when charged by a Daniell cell, as a unit of quantity. However, in spite of this, the committee, led by Thomson,

* "The ohm, the ampère and the volt. A memory of fifty years, 1862–1912," *J. Inst. E. E.* **50**, 560–92 (1913).

† M. Faraday, *Collected Researches*, **1**, para. 807 *et seq.*

‡ Fleming Jenkin, "Report on the new unit of electrical resistance," *Proc. R.S.* **14**, 154–164 (1865).

decided that Weber's proposal was far preferable to the use of any unit of the kind previously described. Can anyone measure the debt of gratitude due to them?

A table, given in the third report of the committee, contains a list of units of resistance which had been discussed, some of which were in actual use. The committee had commenced their work by consulting scientific men on the Continent and in America as to the system of units to be adopted, and, as a result of the discussion which followed the receipt of the replies, arrived at a provisional conclusion as to what would be the best system of units to adopt. Their first report (Cambridge, 1862) gives their reasons for the adoption of the absolute system based on the centimetre, the gramme and the second, and in their second report, 1863, will be found the celebrated appendix C* by J. Clerk Maxwell and Fleming Jenkin which explained, once for all and in a manner in which the experience of seventy years has found no flaw, the elementary relations between electrical measurements.

No doubt all electricians to-day know these relations almost by instinct, but is it too much to hope that this lecture may induce some who have never done so to go back to this appendix† and study the words Clerk Maxwell used to commend the c.g.s. system?

But the establishment of a system of units theoretically sound did not constitute the whole work of the committee. There was a second task, that of improving the construction of practical standards for electrical measurements. The ohm, the new unit of resistance, could be reproduced in material form as the resistance of a coil of wire or a column of mercury; such standards had been constructed by Prof. Weber, and in their second report (Newcastle-on-Tyne, 1863) the committee described experiments by Thomson's method of the revolving coil‡, carried out at King's College by Maxwell, Balfour Stewart and Fleming Jenkin, for the measurement of electrical resistance. The result was expressed as the resistance of a coil of wire known as "June 4" which the committee found to have a resistance of 107,620,116 m./sec. The revolving coil apparatus is among the exhibits on the table.

Meanwhile Matthiessen had been at work endeavouring to find the alloy best suited for the construction of permanent standard of resistance, while during 1863 a further set of measurements were made with the revolving coil. These are described in the report for 1864. Platinum silver in the proportion of 66 per cent.

* This appendix is called B in the index to the report, and C in the main report; there is no appendix B.

† This appendix and the whole of the reports of the committee will be found in the collected volume of British Association reports on electrical standards edited in 1913 by F. E. Smith as a record of the history of absolute units and of Lord Kelvin's work in connexion with these—a memorial volume made possible by the generosity of Mr R. K. Gray and the Council of the Association.

The following were the members of the committee from its commencement in 1861 to 1870, when it ceased its work temporarily: Lord Kelvin, Prof. A. Williamson, Sir Charles Wheatstone, Prof. W. H. Miller, Dr A. Matthiessen, Prof. Fleming Jenkin, Mr C. F. Varley, Prof. Balfour Stewart, Mr C. W. Siemens, Prof. J. Clerk Maxwell, Dr Joule, Sir Chas. Bright, Dr Esselbach, Prof. G. C. Foster, Mr Latimer Clark, Mr D. Forbes and Mr Ch. Hockin.

‡ The method had been indicated a few months previously by Weber in his paper "Zur Galvanometrie," but the committee appear not to have been aware of this.

Table 5. Resistance standards in use in 1863.

Description	Name	Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$	Observations
Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$ electromagnetic units (new determination)	Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$	1·000	Calculated from the B.A. unit
Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$ electromagnetic units (old determination)	Thomson's unit	1·0505	From an old determination by Weber
Twenty-five feet of a certain copper wire, weighing 345 grains	Jacobi	1·988	No measurement made; ratio between Siemens (Berlin) and Jacobi taken from Weber's *Galvanometrie*
Absolute $\frac{\text{metre}}{\text{second}} \times 10^7$ electromagnetic units determined by Weber (1862)	Weber's absolute $\frac{\text{metre}}{\text{second}} \times 10^7$	3·015	Measurement taken from a determination in 1862 of a standard sent by Prof. Thomson; does not agree with Weber's own measurement of Siemens's units; by Weber 1 Siemens's unit = 1·025 × 10^7 metres-second
One metre of pure mercury, one square millimetre section, at 0° C.	Siemens 1864 issue	3·138	Measurement taken from three coils issued by Messrs Siemens
One metre of pure mercury, one square millimetre section, at 0° C.	Siemens (Berlin)	3·156	Measurement taken from coils exhibited in 1862 by Messrs Siemens, Halske & Co. (well adjusted)
One metre of pure mercury, one square millimetre section, at 0° C.	Siemens (London)	3·194	Measurement taken from coils exhibited in 1862 by Messrs Siemens, Halske & Co. (well adjusted)
British-Association unit	B.A. unit, or Ohmad	3·821	Equal to 10,000,000 $\frac{\text{metres}}{\text{second}}$, according to experiments of Standard Committee
One kilometre of iron wire, four millimetres in diameter (temperature not known)	Digney	30·40	From coils exhibited in 1862 (pretty well adjusted)
One kilometre of iron wire, four millimetres in diameter (temperature not known)	Bréquet	32·03	From coils exhibited in 1862 (indifferently adjusted)
One kilometre of iron wire, four millimetres in diameter (temperature not known)	Swiss	34·21	From coils exhibited in 1862 (badly adjusted)
One English standard mile of pure annealed copper wire $\frac{1}{16}$ in. diameter at 15·5° C.	Matthiessen	44·57	From a coil lent by Dr Matthiessen (of German-silver wire)
One English standard mile of one special copper wire $\frac{1}{16}$ in. in diameter	Varley	84·01	From coils lent by Mr Varley (well adjusted)
One German mile = 8238 yards of iron wire $\frac{1}{8}$ in. in diameter (temperature not known)	German mile	188·4	From coils exhibited in 1862 by Messrs Siemens, Halske & Co.

silver to 33 per cent. platinum was chosen as the metal for standard coils, but resistances of 1 B.A. unit were constructed in platinum and a number of other alloys. Some of these are exhibited. The first series of comparisons of B.A. units to be deposited at Kew Observatory was made by Hockin and reported on in the fourth report (Birmingham, 1865). This is shown in table 6.

Table 6. Comparison of B.A. units to be deposited at Kew Observatory, by C. Hockin.

Material of coil	Number of coil	Date of observation	Temperatures at which coil has a resistance of 107 m./sec. (° C.)	Observer
Platinum-iridium alloy	2	January 4, 1865	15·5	C. H.
		June 6, 1865	16·0	A. M.
		February 10, 1867	16·0	C. H.
Platinum-iridium alloy	3	January 4, 1865	15·3	C. H.
		June 6, 1865	15·8	A. M.
		February 10, 1867	15·8	C. H.
Gold-silver alloy	10	January 5, 1865	15·6	A. M.
		February 10, 1867	15·6	C. H.
Gold-silver alloy	58	April 10, 1865	15·3	A. M.
		June 6, 1865	15·3	A. M.
		February 10, 1867	15·3	C. H.
Platinum	35	January 7, 1865	15·7	C. H.
		August 18, 1866	15·7	A. M.
		February 10, 1867	15·7	C. H.
Platinum	36	January 7, 1865	15·5	C. H.
		August 18, 1866	15·5	A. M.
		February 10, 1867	15·7	C. H.
Platinum-silver alloy	43	February 15, 1865	15·2	C. H.
		March 9, 1865	15·2	A. M.
		February 10, 1867	15·2	C. H.
Mercury	I	February 2, 1865	16·0	A. M.
		July 18, 1866	16·0	A. M.
		February 11, 1867	16·7	C. H.
Mercury	II	February 3, 1865	14·8	A. M.
		August 18, 1866	14·8	A. M.
		February 11, 1867	14·8	C. H.
Mercury	III	February 11, 1867	17·9	C. H.

The fifth report (Dundee, 1867) contains Thomson's report on electrometers and electrostatic measurements, in which among others the absolute electrometer (one is exhibited) was described. I began work at the Cavendish Laboratory in 1876, and this was the apparatus which I was set to use in order to check the e.m.f. of a series of tray Daniell cells, employed by Chrystal in his experiments on Ohm's law. The report also contains Joule's determination of the dynamical equivalent of heat from the thermal effects of electric currents, in which he described for the first time the use of a balance for measurement of current.

In 1870 the committee expressed the view that the work could be better advanced if they were not reappointed, but three new committees of smaller numbers were chosen instead to determine and issue (1) a condenser representing the unit of capacity, (2) a gauge for showing the unit difference of potential, (3) an electrodynamometer adapted to measure the intensity of currents in a decimal multiple of the absolute measure. The committee was not at once reappointed, and the small committees were not set up.

Meanwhile, doubts had been thrown on the accuracy of the result found by the committee for the value of the ohm, both by the experiments of Joule already referred to and by investigations in Germany by Kohlrausch and H. Weber and in America by Rowland.

The B.A. standard resistances had been transferred by Maxwell from Kew to the Cavendish Laboratory, and compared by Chrystal and Saunder in 1876. Between 1878 and 1881 a further comparison was instituted by Fleming. The apparatus he used is on the table. It was described* in a paper read to this Society in 1880. His results were expressed in a series of curves giving the value of the coils at various temperatures in terms of the mean B.A. unit, defined as the mean of the values of the seven coils tested by Hockin at the temperatures at which each was stated to be correct. Fleming's chart was for many years the standard to which all resistances were referred.

Lord Rayleigh, appointed Cavendish Professor in 1879, soon became interested in electrical measurements; the need for standards was becoming felt by engineers; and Ayrton in 1880 at the Southampton meeting of the B.A. moved for the reconstitution of the committee. The new committee made its first report in 1881 at the jubilee meeting of the Association at York. At this meeting I became a member of the Association; and shortly afterwards Lord Rayleigh asked me to help in the issue of certificates of resistance coils from the Cavendish Laboratory. In the following year I became Secretary of the Committee and closely connected with its work, being in charge of its standards which in 1900 were removed to the National Physical Laboratory.

In 1881 there was an international congress in Paris at which Thomson supported the views of the B.A. committee, and after much discussion the c.g.s. system was accepted as the basis of an international system, while the practical units were to be the ohm, 10^9 c.g.s. units, and the volt, 10^8 c.g.s. units. During the early meetings of the congress Wiedemann and v. Helmholtz had spoken strongly in favour of Siemens's mercury unit as the standard. It was further agreed (1) that the unit of resistance, 1 ohm, should be represented by a column of mercury 1 mm.^{2} in section at the freezing point of water, and (2) that an international commission should be charged with the duty of determining by new absolute measurements the length of this column.

The commission met in 1884. Mascart presented a table giving all the results then known for the length of the mercury column; the mean was 106·02 cm., and the first resolution of the conference stated that "The legal ohm is the resistance

* *Proc. Phys. Soc.* **3** (1880). *Phil. Mag.* **9**, 109 (1880).

of a column of mercury a square millimetre in section and 106 centimetres in length at the temperature of melting ice." The values on which this resolution was based differed among themselves by some 2 per cent. Thomson and the other English representatives pressed for a result more nearly in agreement with the English determinations, with the result that the "legal ohm" never became legal in England.

An interesting relic of this period is on the table in the form of a mercury standard representing the legal ohm constructed by Dr Benoît of the Bureau des Poids et Mesures, and compared with the B.A. units in 1885*.

In 1890 the Board of Trade appointed a committee to report on a standard for use in England, and the upper part of table 7 gives the values then available to guide their decision. As a result the committee adopted as the resistance of the B.A. unit the value 0·9866 ohm and as the length of the mercury column 106·3 cm.

Table 7. Value of the ohm expressed as the resistance of a column of mercury.

Lord Rayleigh	1882	Rotating coil	106·31
Lord Rayleigh	1883	Lorenz	106·27
Mascart	1884	Induced current	106·33
Rowland	1887	Mean of several methods	106·32
Kohlrausch	1887	Damping of magnets	106·32
Glazebrook	1882	Induced currents	106·29
Glazebrook	1888		
Wuilleumier	1888	—	106·31
Jones	1891	Lorenz	106·31
Jones	1892	Lorenz	106·32
Ayrton and Jones	1897	Lorenz	106·27
Guillet	1899	Induced currents	106·21
Campbell	1912	Alternating currents	106·27
		Mean	106·29

The committee also recommended as standards for current and e.m.f. respectively the silver voltameter and Clark's cell, and prepared specifications for their use, basing their recommendation in the main on the work which had been in progress at Cambridge during the previous ten years.

Some of the results on which the decisions were based are shown in tables 8 and 9.

Table 8. Electrochemical equivalent of silver.

		mg./sec.
1884	Rayleigh and Sidgwick	1·1179
1884	Kohlrausch	1·1183
1884	Mascart	1·1156
1890	Pellat and Potier	1·1192
1898	Kahle	1·1183
1898	Patterson and Guthe	1·1192
1903	Pellat and Leduc...	1·1195
1904	Van Dijk and Kunst	1·1182
1906	Guthe	1·1182
1907	Smith, Mather and Lowry	$1·1182_7$
1908	Laporte and de la Gorce	$1·1182_9$
1912	Rosa, Dorsey and Miller	$1·1180_4$

* For a detailed account see *Proc. Phys. Soc.* and *Phil. Mag.* **20**, 343 (1885).

Table 9. E.m.f. of Clark cell at 15° C.

		Volts
1872	Clark	1·4378
1884	Rayleigh and Sidgwick	1·4345
1896	Kahle	1·4322
1899	Carhart and Guthe	1·4333
1905	Guthe	1·4330
1907	Ayrton, Mather and Smith	1·4323
	Indirect determination of e.m.f. of Clark cell	
1884	Von Ettinghausen	1·433$_5$
1885	Rayleigh	1·435
1892	Glazebrook and Skinner	1·434
1904	Trotter	1·432$_9$

The findings of this Board of Trade committee were confirmed at the Edinburgh meeting of the Association in 1891, at which Dr v. Helmholtz was present with Dr Lindeck and Dr Kahler of the Reichsanstalt, M. Guillaume of the Bureau des Poids et Mesures, and Prof. Carhart of the United States. Dr v. Helmholtz expressed his full concurrence in the decisions which were, as he informed the committee, in accord with the recommendations which had already been laid by the Curatorium of the Reichsanstalt, as well as by himself, before the German government. Dr Lindeck laid before the committee information as to the property of the manganese alloy (manganin) used by the Reichsanstalt for resistance coils. Some of the material which was before the committee has been given in tables 7, 8 and 9. An international congress took place at Chicago in August, 1893, at which practically the same words were adopted to define the international standards. In the Chicago resolutions a distinction is drawn for the first time between the c.g.s. units or their multiples and the international standards. Thus the international ohm is said to be based on the ohm of 10^9 c.g.s. units of resistance and to be represented by the resistance of a certain mercury column.

Meanwhile experiments aimed at attaining a higher value of accuracy in all the standards were continued. In 1897 Profs. Ayrton and Viriamu Jones described a new form of Lorenz apparatus, constructed for McGill University, by the aid of which they obtained the value 106·28 for the length of the mercury column having a resistance of one ohm, and next year the same authors described an ampere balance of high accuracy. The National Physical Laboratory was opened at Teddington in 1902, and the resistance coils and other standards came under the charge of F. E. Smith, now Sir Frank Smith. Arrangements were made next year to set up an ampere balance in accordance with the Ayrton Jones designs, and a series of papers appeared in 1907 giving an account of work by Ayrton, Makin, Smith and Lowry on the current balance, the silver voltameter and the Weston cell. This cell had been recommended at a conference held at the Reichsanstalt in 1905 in place of the Clark cell. As a result of discussions at St Louis in 1904 and the Berlin Conference, it was decided to hold an International conference in London in 1908,

and this duly took place. Lord Rayleigh was chairman and Mr Duddell acted as secretary.

After much discussion agreement was reached as to the definitions of the ohm, the ampere and the volt, multiples of the c.g.s. units of resistance, current and electromotive force, and the international standards by which these units were to be represented. I will refer later to the reasons for this distinction.

The conference was attended by representatives of twenty-five countries, who all signed its report. It also set up an international committee to formulate a plan for, and to direct, such work as may be necessary in connexion with the maintenance of standards, etc. As a result of this, representatives of the N.P.L., the Reichsanstalt and the Laboratoire Centrale d'Électricité, by the kindness of Dr Stratton, met at Washington in 1910 and conducted a valuable series of comparisons. Since that date close co-operation has been maintained between the four laboratories mentioned.

Meanwhile the work of realizing the c.g.s. units to the highest accuracy has continued. In 1902, by the generosity of the Drapers' Company and of Sir Andrew Noble, funds had been found for the construction at the N.P.L. of a greatly improved form of Lorenz apparatus, and the results of Smith's measurements with this apparatus were published in 1914*. He also set up a number of mercury resistance tubes, and as a result found for the length of the mercury column having a resistance of one ohm the value 106·27 cm.

Figures 5 and 6 show the Lorenz apparatus at the N.P.L. and the Ayrton Jones current balance, while in table 10 will be found the results of the more recent determinations of the international ohm, ampere, volt and watt.

Table 10. Values of international units (1831) in terms of c.g.s. units.

International ohm	$(1{\cdot}00051 \pm 0{\cdot}00002)\ 10^{9}$
,, ampere	$(1 - 0{\cdot}00006 \pm 0{\cdot}00006)\ 10^{-1}$
,, volt	$(1{\cdot}00045 \pm 0{\cdot}00008)\ 10^{8}$
,, watt	$(1{\cdot}00039 \pm 0{\cdot}00014)\ 10^{7}$
E.m.f. of Weston normal cell at 20° C.	$1{\cdot}01876 \times 10^{8}$ c.g.s. units

Intercomparisons of the original B.A. coils have been continued from time to time, and a very complete discussion by F. E. Smith of the changes in their values will be found in the report for 1908. The coils were compared with the resistance of mercury by myself in 1888 and by Mr Smith twenty years later in 1908. On the assumption that the resistance of 1 m. of mercury is 0·95352 B.A.U., the value given by my experiments, we obtain the results given in table 11. These results being accepted, it would appear that the two coils of pure platinum D and E did not change during that period.

Thus, the differences between the various coils being known, it becomes possible to form a table giving the values of the coils in terms of the original B.A.U. at each of the periods at which complete intercomparisons were made. This is done in table 12.

* F. E. Smith, *Phil. Trans. R.S.* **214** A, 27 (1914).

Table 11. Values of coils at 16·0° C. in 1888 and 1908 obtained from comparison with mercury tubes, the resistance of 1 m. of mercury being assumed to be 0·95352 B.A.U.

Coil	Value in 1888 at time of determination of specific resistance of mercury	Value in 1908
A	1·00068	1·00042
B	1·00025	1·00018
C	1·00067	1·00093
D	1·00013	1·00012
E	1·00073	1·00072
F	0·99970	1·00080
G	0·99936	1·00095
H	0·99963	0·99964
Flat	1·00023	1·00045

Table 12. Resistances at 16·0° C. in terms of the original B.A.U. (1867). Values obtained through the two platinum coils D, E.

Coil	Material	1867	1876	1879–81	1888	1908
A	PtIr	1·00000	1·00077	1·00056	1·00147	1·00122
B	PtIr	1·00029	1·00121	1·00080	1·00104	1·00098
C	AuAg	1·00050	1·00141	1·00101	1·00146	1·00173
D	Pt	1·00092	1·00092	1·00092	1·00092	1·00092
E	Pt	1·00152	1·00152	1·00152	1·00152	1·00152
F	PtAg	—	—	1·00016	1·00072	1·00160
G	PtAg	1·00022	1·00030	0·99982	1·00025	1·00175
H	PtAg	1·00020	—	—	1·00042	1·00044
Flat	PtAg	—	—	1·00079	1·00120	1·00125

§ 9. FUNDAMENTAL INTERNATIONAL ELECTRICAL STANDARDS

And here it is desirable to include a brief notice of recent action taken by the International Committee on Weights and Measures with regard to electrical standards and approved by the International Conference of Weights and Measures in 1930.

Some forty years after England had accepted the metric convention there was an important change. In pre-war days there had been discussions as to an extension of the functions of the International Committee of Weights and Measures. Sir David Gill and Dr Stratton, then Director of the Bureau of Standards, had taken a prominent part in these, but it was not until 1927 that action was taken. The following resolution, ratified later by the Seventh International Conference on Standards, was approved on October 4 of that year: "The International Committee of Weights and Measures approves the organisation of a Consultative Committee for Electricity (Comité consultatif d'Électricité) to advise the International Committee of Weights and Measures on questions relating to systems of Measurement and Electrical

Standards." The first meeting of this* committee was held in November 1928, and the information which follows is taken from the proceedings† of that meeting.

After considering the question of the unit‡ to be employed, the committee proceeded to discuss the advice they should give to the International Committee of Weights and Measures as to electrical units. In this connexion it is necessary to bear carefully in mind the distinction between electrical standards and those of length and mass already in the care of the Bureau. For length and mass there are material standards assumed to be invariable in form and dimensions. These have to be maintained in safety and used from under carefully defined conditions for intercomparison with the national standards of the countries adhering to the convention. With electrical standards no such simple course is possible. No permanent standard of resistance or electromotive force has been found. Changes, some of them no doubt very slight, have occurred in all known resistance coils§, while no voltaic cell preserves its electromotive force unaltered. It is not possible therefore to equip the Bureau with invariable standards against which national standards could be compared. Recourse must of necessity be had to experimental determinations of the standards in terms of the absolute units of length, mass and time. This requires expensive apparatus and much skilled experimenting. Again, all such work is liable to error; it is desirable therefore that it should be carried out in a number of laboratories under various conditions.

Now the national and other laboratories of several countries are already equipped for such work, and the staff have experience in the measurements. It was therefore thought right by the committee to utilize the opportunities and experience thus afforded, and not to plan at the Bureau the extensive and elaborate equipment necessary if the work of determining *ab initio* international standards for electricity. Instead of this the national laboratories are invited to continue their work and to send the result of their investigations to the Bureau in such form as the committee may desire. These results will be co-ordinated by the committee, and international standards will be formulated in accordance with the combined results.

And thus the Comité consultatif d'Électricité agreed to advise the International

* The Members were: President: M. le Sénateur Volterra, President of the International Committee. Deputy President: M. Paul Janet, Director of the Laboratoire Central d'Électricité. Members: for the Reichsanstalt, Berlin, Prof. Steinwehr; for the Bureau of Standards, Dr Geo. R. Burgess, Director of the Bureau; for the National Physical Laboratory, Dr D. N. Dye, F.R.S.; for the Laboratoire Central d'Électricité, M. R. Jouast, Deputy Director; for the Electrotechnical Laboratory of the Royal College of Engineers, Rome, Prof. L. Lombardi, Director; for the Electrotechnical Laboratory, Tokyo, M. Seikighi Jimbo, Assistant; for the Central Chamber of Weights and Measures, Leningrad, Prof. D. Konorator, President; for the Bureau International des Poids et Mesures, Sèvres, M. Ch. Éd. Guillaume, Director.

† Comité consultatif d'électricité, *Rapport Procès-verbaux des Séances de* 1928.

‡ Some results of this discussion are dealt with later. See pp. 452 *et seqq.*

§ Among the most permanent of these are the two platinum B.A. coils D and E, constructed in 1863, for which the following values have been found by comparison with mercury tubes:

	1888	1908
D	1·00013	1·00012
E	1·00073	1·00072

Committee that the functions entrusted to the International Bureau of Weights and Measures in regard to electrical units should be to establish: (1) A central secretariat to organize the systematic exchange of standards (étalons) and secure the co-ordination (assurer la synthèse) of the results of comparisons made by the national laboratories. (2) A laboratory to which material standards representing the results obtained in different countries can be set for exact comparisons. (3) A collection of reference standards and working standards, including standards of inductance and capacity, together with the equipment necessary for the comparison of other standards with those of the bureau. It is pointed out that such a scheme places very definite responsibilities on the national laboratories, and the report continues with regard to the

Functions of the International Committee. "If the general plan be approved* the International Committee under the authority given to it by the General Conference will have the responsibility of fixing and promulgating the values to be employed for practical standards and of fixing the date of any fresh revision. For this purpose the Consultative Committee can continue to advise the International Committee in accordance with the functions which the General Conference has assigned to it.

The Report continues: "It is to be noted that the plan does not envisage the establishment in any one laboratory of standards to be regarded as representative of the electrical units or as having an authority superior to that of the standards of other laboratories. The essential point is that the electrical units are considered as secondary, in the sense that they are based directly on the fundamental units of length, mass and time. It is common knowledge that material electrical standards change more or less in course of time. The determination of their exact value in terms of the fundamental units, the maintenance of the standards themselves in the highest degree of precision possible, and the development of means certain to secure their constancy are problems which necessitate the combined resources of the best laboratories of the world. While a part of this task could be undertaken under the auspices of the International Committee in the laboratory of the Bureau International, the Committee and the Bureau can do a more useful work by combining the results of research made throughout the world.

"As the result of the authority given to it to co-ordinate electrical measurements the International Committee has ample power to carry into effect this plan."

The above extract makes clear the conditions and method under which the International Committee of Weights and Measures has undertaken electrical standardization.

§ 10. NATURAL AND ARBITRARY STANDARDS

If we look back over the history of the past two hundred years or so we may note some interesting changes in the views of scientific men as to the bases on which to rest their standards of measurement.

If we go to a much earlier period it is obvious that weights and measures began with some natural objects selected for their convenience, and sufficiently constant

* The plan was approved at the meeting in 1931.

in quantity for the simple needs of the times. Such were the length of a man's forearm or the weight of corn—possibly of water—contained in a vessel approximately constant in volume.

In time, uniformity became more important and so we pass from the length of the king's arm to the scale engraved on the wall or floor of the cathedral, and from the weight of so many grains of wheat taken from the middle of the ear to the mass of a lump of metal.

The jury of Queen Elizabeth's time, to which we owe our modern system, paid little attention, it is probable, to the origin of the yard or the pound; they fixed on the length of a metal bar and the weight of a lump of iron and prescribed that these were to be the standards, trusting to the permanence of these material objects rather than to any natural laws which might serve for their definition. They were no doubt severely practical; a change came in England, in the early days of the Royal Society, when we find Caswell and Walker endeavouring to connect the weight and volume of a quantity of water and thus establish a natural standard of weight in terms of the standard of length, trying possibly to repeat what had been done in Egypt long ago.

Then some fifty years later Graham, realizing that there was a definite relation between the period of a body in oscillation, its dimensions, and the distribution of its mass, endeavoured to connect the period of a seconds pendulum with its length. He set about the comparison of existing material standards, to find one in terms of which to express the length of the pendulum, thus leading to the report of Lord Macclesfield's committee in 1743.

Intercomparisons between existing standards continued to be made, and the idea of basing that of length on the seconds pendulum increased in favour until in 1814 the House of Commons asked for a formal statement of the length, and two years later the Astronomer Royal was instructed to determine it. Kater's work followed, with the result that, while in 1824 certain material standards—Bird's bar of 1760 and a brass weight of 1 lb. Troy dated 1758—were made the legal standards, they were to be recovered if lost by reference to natural standards, the length of the seconds pendulum and the weight of a cubic inch of water.

But this recourse to natural standards did not last long. Scientific men discovered that it was simpler to compare a standard with its copies, and to recover its value if lost by the aid of those copies, than to determine the length of the seconds pendulum or the mass of a cubic inch of water, and so in 1843 the recommendations of 1824 were thrown over; the Scientific Committee which had been appointed in 1838 to advise, recommended that the standards which had been lost in the fire of 1834 should be recovered from their copies "and that the standard be in no way defined by reference to any natural basis." More trust could be placed in the permanence of a material standard and on the accuracy with which it could be recovered by intercomparison of its copies, than on the result of any attempt to reconstruct it based on some natural law, and so it has remained up to the present.

In France the history has been the same. The Constituent Assembly entrusted

the Academy of Science with the duty of introducing a new system of measures, and in 1788 a report* was issued on the choice of a unit of measure drawn up by Borda, Lagrange, Laplace, Mouge and Condorcet. The report discussed the advantages of employing either the length of the seconds pendulum or that of a meridian of the earth as the basis of length-measurement and decided in favour of the latter which, in the words of Laplace, "appears to be of very high antiquity, it is so natural to man to refer measures of distance to the dimensions of the globe which he inhabits, in order that in transporting himself from place to place he may know, by the denomination of the space passed through alone, the relation of this space to the entire circumference of the earth. This method has also the advantage of making nautical measures correspond at once with celestial ones." And so the metric system came into being. Delambre† and Mechain measured the length of an arc from Dunkirk to Barcelona and hence calculated the length of the quadrant as 5,130,740 of a certain iron toise when at a temperature of $61\frac{1}{2}^{\circ}$ F. The metre was fixed at one ten-millionth of this length, and a standard metre constructed to represent it, while the kilogramme was to be the mass of a cubic decimetre of water at the temperature of its maximum density and a platinum weight was constructed with great care to have this mass when weighed *in vacuo*.

But, while the material standards continue to exist, research has shown that they no longer represent the definitions. The metre is still the distance between the marks on a certain bar of platinum-iridium but the earth's quadrant is not 10^7 of these metres, while different meridians differ in length; the earth is not a spheroid. The best mean figure obtainable seems to be 1 meridional earth quadrant = 10,002,090 metres.

The platinum mass, known as the Kilogramme des Archives, is still the standard kilogramme, but the mass of a cubic decimetre of water at 4° C. approximately has been found to be 999·972 gm.; or alternatively 1000·028 cm.^{3} is the volume of a kilogramme of water at the temperature of its maximum density and under normal atmospheric pressure.

Neither the seconds pendulum nor a quadrant of the earth has proved to be more satisfactory than the material standard. This was the case when Michelson‡ in 1889, realizing that the wave-length of light of a given frequency was invariable, suggested its use as an ultimate standard of length. There was the difficulty of stepping from so small a unit as a wave-length up to the metre, but he pointed out that one can bridge over the distance by using a number of standards each approximately, but not quite, double the distance of the last, employing interference methods to determine the difference between the length of the larger standard and twice that of the smaller. Light arranged to traverse the smaller distance twice is made to interfere with light which has only once traversed the longer distance. Michelson§ had an opportunity of carrying this idea into practice somewhat later

* *Mém. de l'Académie* (Paris, 1788 H 7–17).

† Delambre, *Base du Système Métrique* (3 vols. 4to. Paris).

‡ Michelson and Morley, *Am. J. of Sci.* **38** (1889).

§ *Travaux et Mémoires du Bureau International des Poids et Mesures*, **9** (1895).

at the Bureau des Poids et Mesures at Sèvres, and found from observations on the red line of cadmium that its wave-length λ_R is given by

$$\lambda_R = 0{\cdot}64384722 \times 10^{-6} \text{ m.}$$

Somewhat later, in 1913, Messrs Benoît, Fabry and Perot* at the Bureau International repeated the measurement, using a modification of the method, and found

$$\lambda_R = 0{\cdot}6438470 \times 10^{-6} \text{ m.}$$

Since that date measurements have been made in Japan by Messrs Watanambe and Imaizumi leading to the result

$$\lambda_R = 0{\cdot}64384685 \times 10^{-6} \text{ m.,}$$

while Dr Tutton communicated the results of his experiments to the Royal Society on April 3 last: according to these he finds

$$\lambda_R = 0{\cdot}64384698 \times 10^{-6} \text{ m.}$$

Mr Sears gives as a provisional result of his N.P.L. experiment

$$\lambda_R = 0{\cdot}64384714 \times 10^{-6} \text{ m.}$$

At their meeting in 1925 the International Committee of Weights and Measures agreed in principle to define the unit of length by means of the wave-length of light, subject to the determination by the national laboratories of the most satisfactory method of realization. As a result, work has been in progress at the National Physical Laboratory for some time and now is approaching conclusion, while work is in progress at the Reichsanstalt and at the Bureau of Standards. I am indebted to Mr Sears for the following description of the investigation which is being carried out at Teddington by Mr Barrell.

"The apparatus which has been constructed for this purpose is based on the method of Fabry and Perot in which the separation of two parallel semi-silvered glass plates, constituting the first étalon, is determined directly in monochromatic light by the method of coincidences, and then compared by the superposition of fringes in white light with that of similar longer étalons which are very closely but not quite exactly multiples of the first. The étalons are set up in line, and the light passes through them in succession. In the N.P.L. apparatus the difference from the exact multiple is determined by the tilting of one of the étalons through a measured angle on either side of the normal until the optical path in the longer étalon is an exact multiple of that in the shorter. The étalons take the form of invar tubes whose ends are chromium-plated and polished optically flat and parallel, the glass plates being wrung on to these surfaces. It is thus possible to evacuate the tubes and work with wave-lengths *in vacuo*, or alternatively to work in air in any desired controlled condition. The longest étalon is slightly greater than 1 m. in length, and contains a steel end-gauge of X-shaped cross-section, 1 m. in length, whose ends also are polished flat and square to its axis. The small distances between the ends of this gauge and the glass terminal plates of the étalon are determined directly by interference in monochromatic light, while the length of the étalon is determined by

* *Travaux et Mémoires*, 15 (1913).

means of light passing along the four channels formed between the arms of the X and the wall of the tube. By subtraction the length of the metre bar is finally ascertained in terms of the chosen wave-length of monochromatic light—usually the cadmium red.

"The apparatus is set up in a carefully designed case which is thermally controlled by electrical heaters governed by a thermostat. Air is circulated over these heaters and through the enclosure by means of a fan and ducts leading to rotating cowls, which vary the direction of flow and so effect complete mixing and stirring. The temperature of the whole enclosure can be maintained constant throughout to within 0·01° C. over long periods of time. On the outside of the tube of the longest étalon is wound a platinum resistance thermometer by means of which its temperature can be ascertained at any time to 0·001° C. A very high accuracy of temperature-measurement is necessary, as it is hoped that the apparatus will be capable of repeating measurements to an accuracy of the order of 1 part in 40,000,000, which is about ten times as great as can be obtained with certainty in the microscopic comparison of line standards."

Thus it will be seen that the International Committee are prepared to refer the standard metre to a natural standard against which it can be compared. This natural standard will be represented by the distance between two marks on a bar of platinum-iridium, and the constancy of this distance will be checked from time to time by comparison with the wave-length of light of definite period. Hence there is a prospect that a natural standard of length will in time be generally accepted.

As to a standard of mass, up to the present it is not probable that the kilogramme, the mass of a lump of platinum, will be disturbed until we reach the condition when the mass of (say) the atom of hydrogen or of an electron can be determined with sufficient accuracy.

For time we already have a natural standard in the sidereal day, and, while the frequency of a quartz or other oscillator will certainly be used for comparatively short intervals, it will not replace the day and hour for longer periods.

In electrical standards also a transference to natural standards is taking place.

One of the first duties of the Comité Consultatif d'Électricité set up in 1927 was, as has been stated, to determine the units in terms of which electromotive force, current and resistance are to be measured. At present, while the units are defined in terms of the c.g.s. units of length, mass and time, the international standards used in daily work are, for the ohm 10^9 c.g.s. units, the resistance of a column of mercury; for the volt 10^8 c.g.s. units, the e.m.f. of a Weston cell; and for the ampere 10^{-1} c.g.s. units, the current which deposits a certain weight of silver per second in a voltameter of specified design. This weight of silver, it should be noted, has been determined by the weighing in an ampere balance of the electromagnetic force set up by a current of one ampere. These definitions, as has already been explained, were agreed to at the international conference in London in 1908.

The appliances needed for an absolute determination of the electrical units are elaborate and expensive. At the time of the London conference it was realized that the necessary measurements could be made only in a few well-equipped standardizing

laboratories, while it was felt that the standards for daily use should be such as could be set up without serious difficulty in any laboratory provided with ordinary measuring and weighing apparatus of adequate precision. The arrangements proposed at Sèvres by the Comité Consultatif d'Électricité have altered this. When these are complete any country can have its electrical standards compared against international standards maintained at the Bureau, while by the collaboration of countries possessing national laboratories, equipped for absolute measurements, the international standards of the Bureau can be realized and maintained in much closer agreement with the c.g.s. units than is now the case with the international ohm, volt, and ampere. Dr Dye tells me that at present:

One international ohm (as defined) = $(1{\cdot}00051 \pm 0{\cdot}00002)\ 10^9$ c.g.s. units.

For an actual resistance coil the uncertainty must be increased to $0{\cdot}00004 \times 10^9$. As to the ampere the figures cannot be quite so definite, for no absolute determination of the ampere has been made for a number of years, and the comparisons which have been made all assume the "constancy of standard cells of which there is definite disproof in the international comparisons of late years." He gives, however, as the best result available the relation

10^{-1} c.g.s. units of current = $(1{\cdot}00006 \pm 0{\cdot}00006)$ international ampere.

And so the Comité Consultatif at its meeting in 1928 adopted the following resolutions. "(1) Le Comité consultatif d'Électricité institué auprès du Comité international des Poids et Mesures, considérant la grande importance qu'il y a à unifier les systèmes de mesures électriques sur une base dépourvue de tout caractère arbitraire, reconnaît dès sa première réunion que le système absolu, dérivé du système c.g.s., pourra être avec avantage substitué au système des unités internationales pour toutes les déterminations scientifiques et industrielles, et décide d'en proposer l'adoption au Comité international des Poids et Mesures. (2) Le Comité consultatif d'Électricité, tout en reconnaissant les grands progrès déjà accomplis dans le domaine des mesures électriques de haute précision, ne croit cependant pas qu'il soit possible dès maintenant de fixer avec toute l'exactitude nécessaire et dont ils sont susceptibles les rapports qui existent entre les unités absolues dérivées du système c.g.s. et les unités internationales de courant de force électromotrice et de résistance telles qu'elles ont été définies par le Congrès international de Chicago en 1893 et la Conférence de Londres en 1908, et émet le vœu que des recherches soient poursuivies dans ce but dans les laboratoires convenablement outillés suivant un programme préalablement étudié en accord avec le Comité consultatif d'Électricité."

The method by which the results obtained in the various laboratories are to be utilized by the Bureau International has already been referred to.

Since 1928, and indeed previously to that date, work has been in progress at the National Physical Laboratory in connexion with all the units. The ampère balance has been reconstituted, work has been in progress on a novel method of measuring a resistance suggested some years since by Albert Campbell, preparations are going

forward for further measurements with the Viriamu Jones Memorial Lorenz apparatus and for a redetermination of the electromotive force of a Weston cell.

At the second meeting of the Comité Consultatif d'Électricité in 1930, reports as to progress were received from the N.P.L., the Bureau of Standards, Washington, the Central Chamber of Weights and Measures of Russia, and the Electrotechnical Laboratory of Tokyo.

While, of course, it will be necessary to have at Sèvres material standards or apparatus embodying the results of these various absolute measurements, these standards and apparatus will represent, with all the accuracy possible, multiples of the c.g.s. units themselves. They will not be material standards adopted because at the time they were constructed they were believed to represent the theoretical units to the highest accuracy then possible. Thus in electricity we go back through the c.g.s. units to the natural standards of length and time, the wave-length of the red line of cadmium and the sidereal second.

§ 12. ACKNOWLEDGMENTS

I cannot conclude without expressing my indebtedness to many friends, and first and foremost to Sir Henry Lyons for the use of the Lecture room and most of the historic apparatus shown on the table, to the Deputy Warden of the Standards for apparatus in his charge, then to the Director of the N.P.L. and Messrs Sears and Dye for their valued help and to Mr Christelow for his care in preparing the slides, also to Mr Hope Jones for the slide of the Shortt Clock. For the part relating to ancient standards thanks are due to Col. N. Belaiew, C.B., to Mr Sidney Smith of the British Museum and to the Library Staff of the Royal Society who helped in looking up the references.

APPENDIX 1

Report of the Royal Commission of 1818

Secretary, Dr Thos. Young, F.R.S.

First Report.

I. Upon a deliberate consideration of the whole of the system at present existing, we are impressed with a sense of the great difficulty of effecting any radical changes, to so considerable an extent as might in some respects be desirable; and we therefore wish to proceed with great caution in the suggestions which we shall venture to propose.

II. With respect to the actual magnitude of the standards of length, it does not appear to us that there can be any sufficient reason for altering those which are at present generally employed. There is no practical advantage in having a quantity commensurable to any original quantity existing or which may be imagined to exist in nature, except as affording some little encouragement to its common adoption by neighbouring nations. But it is scarcely possible that the departure from a standard, once universally established in a

great country, should not produce much more labour and inconvenience in its internal relations than it could ever be expected to save in the operations of foreign commerce and correspondence, which always are and always must be conducted by persons to whom the difficulty of calculation is comparatively inconsiderable, and who are also remunerated for their trouble, either by the profits of their commercial concerns, or by the credit of their scientific acquirements.

III. The subdivisions of weights and measures at present employed in this country appear to be far more convenient for practical purposes than the decimal scale, which might perhaps be preferred by some persons for making calculations with quantities already determined. But the power of expressing a third, a fourth, and a sixth of a foot in inches without a fraction, is a peculiar advantage in the duodecimal scale: and, for the operation of weighing and measuring capacities, the continual division by two renders it practicable to make up any given quantity with the smallest possible number of standard weights or measures, and is far preferable in this respect to any decimal scale. We would therefore recommend that all the multiples and subdivisions of the standard to be adopted should retain the same relative proportions to each other as are at present in general use.

IV. The most authentic standards of length which are now in existence being found, upon a minute examination, to vary in a very slight degree from each other, although either of them might be preferred, without any difference that would become sensible in common cases, we beg leave to recommend for the legal determination of the standard yard, that* which was employed by General Roy in the measurement of a base on Hounslow Heath, as a foundation for the trigonometrical operations that have been carried on by the Ordnance throughout the country, and a duplicate of which will probably be laid down on a standard scale by the Committee of the Royal Society appointed for assisting the Astronomer Royal in the determination of the length of the pendulum; the temperature being supposed to be 62° of Fahrenheit when the scale is employed.

V. We propose also, upon the authority of the experiments made by the Committee of the Royal Society, that it should be declared for the purpose of identifying or recovering the length of this standard, in case that it should ever be lost or impaired, that the length of a pendulum vibrating seconds of mean solar time in London, in the level of the sea, and in a vacuum, is 39·1372 inches of this scale; and that the length of the metre employed in France, as the 10,000,000th part of the quadrantal arc of the meridian, has been found equal to 39·3694 inches.

VI. The definitions of measures of capacity are obviously capable of being immediately deduced from their relations to measures of length; but since the readiest practical method of ascertaining the magnitude of any measure of capacity is to weigh the quantity of water which it is capable of containing, it would, in our opinion, be advisable in this instance to invert the more natural order of proceeding, and to define the measures of capacity rather from the weight of the water they are capable of containing, than from their solid content in space. It will therefore be convenient to begin with the definition of the standard of weight; by declaring that nineteen cubic inches of distilled water, at the temperature of 50°, must weigh exactly ten ounces Troy or 4800 grains, and that 7000 such grains make a pound avoirdupois; supposing, however, the cubic inches to relate to the measure of a portion of brass, adjusted by a standard scale of brass. This definition is deduced from some very accurate experiments of the late Sir George Shuckburgh on the weights and measures of Great Britain; but we propose at a future period to repeat such of them as appear to be the most important.

* But see later second report.

VII. The definitions thus established are not calculated to introduce any variation from the existing standards of length and of weight, which may be considered as already well ascertained. But with respect to the measure of capacity, it appears from the Report contained in the Appendix (A), that the legal standards of the highest authority are considerably at variance with each other; the standard gallon, quart, and pint of Queen Elizabeth, which are kept in the Exchequer, having been also apparently employed, almost indiscriminately, for adjusting the measures both of corn and of beer; between which however a difference has gradually, and, as it may be supposed, unintentionally crept into the practice of the excise; the ale gallon being understood to contain about four and half per cent. more than the corn gallon, though we do not find any particular Act of Parliament in which this excess is expressly recognized. We think it right to propose that these measures should again be reduced to their original equality; and at the same time, on account of the great convenience which would be derived from the facility of determining a gallon and its parts, by the operation of weighing a certain quantity of water, amounting to an entire number of pounds and ounces without fractions, we venture strongly to recommend that the standard ale and corn gallon should contain exactly ten pounds avoirdupois of distilled water of 62° Fahrenheit, being nearly equal to 277·2 cubic inches, and agreeing with the standard pint in the Exchequer, which is found to contain exactly 20 ounces of water.

VIII. We presume that very little inconvenience would be felt by the public from the introduction of this gallon in the place of the customary ale gallon of 282 cubic inches, and of the Winchester corn gallon, directed by a Statute of King William to contain 269, and by some later Statutes estimated at $272\frac{1}{4}$ cubic inches; especially when it is considered that the Standards by which the quart and pint beer measures used in London are habitually adjusted, do not at present differ in a sensible degree from the Standard proposed to be rendered general. We apprehend also that the slight excess of the new bushel above the common corn measure would be of the less importance, as the customary measures employed in different parts of Great Britain are almost universally larger than the legal Winchester bushel.

Appendix (*A*). The standards kept at the Exchequer, for the adjustment both of corn and beer measures, are a bushel, a gallon, and a quart, dated 1601, and a pint, dated 1602, all marked with an E. and a crown. They were examined by Sir George Clerk and Dr Wollaston, and the weight of Thames water which they held, at the temperature of 52°, was found as in the subjoined table. Now, since, according to Sir George Shuckburgh's experiments, a cubic inch of distilled water at 60° weighs $252\frac{1}{2}$ grains, the specific gravity of the water being to that of distilled water as 1·00060 to 1 and the apparent specific gravity of distilled water, in a vessel of brass at 52° being to that of water at 62° as 1·00046 to 1, it follows that the apparent specific gravity of the water employed was 1·00106, and that an ounce Avoirdupois corresponded to 1·731 cubic inches. Hence we obtain the contents of the measures in cubic inches, which are compared in the table with the more direct measurement of Mr Bird and Mr Harris reported to the House of Commons in 1758.

	Oz. Avoir.	Cub. in.	Gallon	Rep. 1758
Pint	20·00	34·6 (× 8 =)	276·9	34·8
Quart	40·35	69·8 (× 4 =)	279·3	70·0
Gallon	156·25	270·4 (=)	270·4	271·0
Bushel	1229·85	2128·9 (× $\frac{1}{8}$ =)	266·1	2124·0

The Exchequer standard wine gallon is dated 1707, and was found to contain 133·4 ounces, answering 230·9 cubic inches. An experiment of Dr Wollaston and Mr Carr, 1814, gave 230·8, the mean being 230·85; while the measurement of 1758 made it 231·2. A duplicate of this measure, and of the same date, is kept at Guildhall.

Dr Wollaston and Mr Carr examined also the three other wine gallons at Guildhall. The oldest of these seems to be the same that was measured by Halley and Flamsteed in 1688, and was said to contain 224 cubic inches: its actual capacity is 224·4. The wine gallon of 1773 which is in daily use for adjusting other measures, was probably in the first instance a correct copy of the Exchequer gallon, but has been reduced by a bruise and by the wear of the brim to 230·0 cub. inches, having lost $-\frac{4}{5}$ of a cub. inch or $\frac{1}{300}$ of its whole capacity. The wine gallon of 1798 contains 230·8 cub. inches.

The Excise wine gallon was found by a similar experiment to contain 230·1 cub. inches, having partaken of the progressive deficiency of the Guildhall gallon, from which it was derived."

Second Report.

We have examined, since our last Report, the relation of the best authenticated standards of length at present in existence, to the instruments employed for measuring the base on Hounslow Heath, and in the late trigonometrical operations; but we have very unexpectedly discovered that an error has been committed in the construction of some of these instruments. We are therefore obliged to recur to the originals which they were intended to represent, and we have found reason to prefer the Parliamentary Standard executed by Bird in 1760, which we had not before received, both as being laid down in the most accurate manner, and as the best agreeing with the most extensive comparisons which have been hitherto executed by various observers, and circulated throughout Europe; and, in particular, with the scale employed by the late Sir George Shuckburgh.

We have therefore now to propose that this Standard be considered as the foundation of all legal weights and measures, and that it be declared that the length of a pendulum vibrating seconds in a vacuum on the level of the sea, in London, is 39·13929 inches, and that of the French metre 39·37079 inches, the English standard being employed at 62° of Fahrenheit.

APPENDIX 2

GEORGE HOLT PHYSICS LABORATORY
THE UNIVERSITY OF LIVERPOOL

2nd April, 1931

The bar of wood sent to me and described as an Egyptian cubit is of approximately rectangular section with one corner slightly bevelled.

The ends are rough cut and are not perpendicular to the length nor parallel to each other. Their greatest distance apart (measured along the length) is about 52·7 centimetres, and their least distance about 52·3 centimetres. The bar is subdivided into 7 parts by grooves cut through the bevel, and its *two* adjacent faces, and the middle part is divided into approximately equal segments by a similar groove.

The grooves are not accurately cut, and the distance between adjacent grooves therefore varies appreciably along their length. The last 4 of the 7 parts are each subdivided into 4 segments by grooves cut through the bevel and *one* of its adjacent faces.

The first of the 7 parts is considerably too long, varying from about 7·8 cm. to 7·6 cm. The next five are approximately equal, each varying from about 7·5 cm. to 7·4 or 7·3. The last part is considerably too short, measuring about 7·0 cm.

The 16 segments into which the last 4 parts of the bar are subdivided are very irregular in size, varying from about 2·0 cm. to 1·6 cm., except the last segment of all, which is much too short, varying from 1·5 cm. to 1·3 cm.

(*Signed*) L. R. WILBERFORCE

REVIEWS OF BOOKS

Theoretical Physics, Vol. I, *Mechanics and Heat*, by Prof. W. Wilson, F.R.S. Pp. x + 332. (London: Methuen and Co.) 21*s*.

This is the first of three volumes which together will constitute a systematic treatise on theoretical physics, from Newtonian to the new quantum mechanics. The author's aim is to "present an account of the theoretical side of physics which, without being too elaborate and voluminous, will nevertheless be sufficiently comprehensive to be useful to teachers and students."

So far as can be judged from the single volume now before us, it may be said at once that Prof. Wilson is assured of conspicuous success in this aim, and that he is producing a book which will fill a want which has been long felt by university teachers of physics. Most teachers who have had to organize advanced degree courses have found it difficult to give adequate representation to both the "classical" and the "modern" sides of the subject (to use a distinction which may in itself be objectionable, but is in general pretty well understood). From the whole mass of physical theory and experiment, some form of selection and exclusion principle is imposed by limitations of time and human capacity, and a great deal of the classical physics is unavoidably relegated to the background, in spite of its intrinsic importance and its beauty. If this process is pushed too far, it is apt to have undesirable repercussions—more particularly on the young physicist who is destined for a career in industrial research, where he will learn by daily experience that the older physics has a value quite outside its mere antiquarian interest.

The purpose of this digression is simply to emphasize the service which a work like the one under review can render to students and teachers alike, by providing a single text in which the whole framework of the older and the newer physics is built up in a consecutive and homogeneous form. The treatment is not exhaustive—subjects like capillarity, diffusion and thermal conduction are wholly or largely omitted—but the book as a whole promises to provide just that basis which is required for the building up of a systematic knowledge of the fundamental principles and their more important applications.

There was most emphatically a need for a book of this type, and it is difficult to see how, within its prescribed limits, the present volume could have been improved upon, or how those limits could have been extended without detracting from a main purpose of the work. The author's experience as a teacher is in evidence throughout—not least in his accurate appreciation of the assistance required in purely mathematical difficulties by the average physics student. Many students, by the way, will probably find it instructive to compare his systematic derivations of formulae with the simpler, if less elegant, *ad hoc* proofs which abound in text-books of a more experimental character.

It is a pleasure to come across a book so well written, and one which can be so cordially recommended. The second and third volumes will be awaited with interest—even with some impatience.

H. R. R.

Conférences d'Actualités Scientifiques et Industrielles. (Paris: Hermann et Cie.) (x) "Température des Flammes," by G. RIBAUD. Pp. 43, 5 fr. (xi) "Anistropie des Molécules: Effet Raman," by JEAN CABANNES. Pp. 66, 8 fr. (xv) "Les Statistiques Quantiques et leur Application aux Électrons Libres dans les Métaux," by L. BRILLOUIN. Pp. 44, 5 fr. (xvii) "La Structure et les Mouvements de l'Univers Stellaire," by G. DAMOIS. Pp. 16, 3 fr.

Reports of four sets of interesting lectures, Nos. x, xi, xv, and xvii of the series *Conférences d'Actualités Scientifiques et Industrielles*, have recently come to hand. The first of these (x) is a set of two lectures on the temperature of flames and the radiation of flames and incandescent gases by Prof. G. Ribaud. The most interesting feature of these lectures is the verification that the temperature of the flame can be calculated very closely thermodynamically on the assumption that the gases supplied come (after combustion) adiabatically to thermodynamic equilibrium with the products of reaction. The second (xi) also is a set of two lectures by Prof. J. Cabannes on the anisotropy of molecules and the Raman effect. The author gives a very instructive account of the light that is thrown on the theory of molecular structure by a study of the depolarizing effect in the coherent scattered radiation and by a study of Raman lines. The third (xv) is a lecture by Prof. L. Brillouin on quantum theory statistics and its application to the electronic theory of metals. This gives a clear account of the origin and simple properties of the statistics of Bose-Einstein and Fermi-Dirac, and in the latter statistics, that obeyed by electrons, of the effect of a magnetic field in producing a weak temperature-independent paramagnetism, and of the way in which the properties of a degenerate electron gas suit the requirements of the theory of metals. Finally the last (xvii) is a single lecture by Prof. G. Damois on the structure and movements of the stellar universe. This gives an account of the modern work on the globular clusters and the speculations on the rotation of our own galaxy, a most interesting chapter of physics on the largest conceivable scale. R. H. F.

Acoustics, a Text on Theory and Applications, by G. W. STEWART, Ph.D., and R. B. LINDSAY, Ph.D. Pp. ix + 358. (London: Chapman and Hall.) 25*s*.

In a recent presidential address to the Physical Society, Dr W. H. Eccles dealt with the subject of "The New Acoustics," and incidentally referred to the gap in the literature of the subject subsequent to the publication of the classical works of Rayleigh and Lamb. This gap is being rapidly filled. There are now on the market several good text-books dealing with recent progress in theory, practical measurements, and technical applications. In addition to these text-books there is also a number of books dealing with developments in more specialized branches of the subject. One is therefore surprised to read in the preface of the book under review that "the present work is *unique* in its stress on the important researches of the past decade and in the useful combination of material both from the theoretical and practical viewpoints."

The book is none the less welcome, as a large part of it deals with the original work of the authors. Chapters v, vi, vii and ix, which include sound-transmission through conduits, impedance theory of tubes and horns, filtration of sound, and physiological acoustics, are in this respect of particular interest and importance. In these branches of the subject Dr Stewart has contributed many important original papers which are now collected and summarized in an interesting manner. They will be read carefully by all students. Chapter ix, dealing with physiological acoustics, is unfortunately rather brief. One would have welcomed here more details of Dr Stewart's researches on binaural audition.

On the theoretical side the book fulfils the claims made in the preface, but the same can hardly be said with regard to the practical side. Chapters X (subaqueous sound-ranging and signalling) and XI (architectural acoustics) are examples of this. One cannot fail to notice the omission of references to important work recently carried out in this country. For example, the work of Drs A. H. Davis and E. T. Paris in architectural acoustics is almost completely ignored. In dealing with the methods of measuring absorption-coefficients of materials, the authors refer to Sabine's and Knudsen's full-scale methods only, no consideration being given to the testing of specimens of small area. The section dealing with the echo method of sounding at sea is particularly inadequate. Only a bare reference is made to Langevin's quartz-oscillator method of depth-recording, and no reference at all is made to the echo sounding system developed by the British Admiralty and in regular use on hundreds of ships. This weakness of the book on the practical side is revealed not only in the text but also in the illustrations, which are in almost every case simple line-diagrams to illustrate the theory rather than the method of practical application.

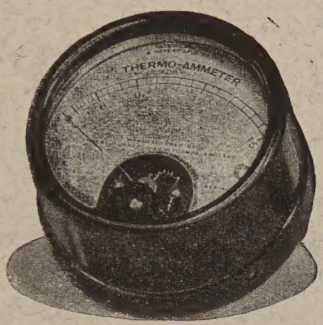
Apart from the description of Dr Stewart's method of measurement of acoustic impedance, Chapter VIII, on acoustic instruments and measurements, is disproportionately short (14 pages). Only a few pages are devoted to intensity-measurement, some of the most important methods being reduced to mere references, and no reference is made to methods of sound-analysis and sound-recording or to measurements of frequency.

In spite of these criticisms, however, the book can be strongly recommended to students as an example of modern tendencies in the theoretical treatment of acoustic problems, the first seven chapters being particularly good in this respect.

The book concludes with an interesting chapter on atmospheric acoustics. This includes a section dealing with the general theory of the propagation of sound through the atmosphere and its application to sound-signalling in air. Tables of data and detailed mathematical processes are given in appendices. The index omits all reference to authors.

A. B. W.

TURNER



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